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USAAVLABS TECHNICAL REPORT 66-8

AN EXPERIMENTAL EVALUATION
OF THE CONFORMAL GEAR

By

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February 1966

U. S. ARMY AVIATION MATERIEL LABORATORIES

FORT EUSTIS, VIRGINIA

CONTRACT DA 44-177-AMC-101(T)
VERTOL DIVISION
THE BOEING COMPANY
MORTON, PENNSYLVANIA

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This report represents a part of a continuing research program for investigations of advanced gear design criteria. The main efforts of this program are directed toward dynamic load tests of specimen conformal contact gearing (Wildhaber-Novikov gear tooth form).

This command concurs with the contractor's conclusions reported herein. The results obtained from this specific study indicate that further research investigations must be conducted before a complete evaluation of conformal contact gearing for aircraft application can be made.

This command concurs with the contractor's recommendations, and the continuing program is scheduled on this basis.

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R-429A

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Fort Eustis, Virginia

SUMMARY

This report concludes Phase II of a study on a new or improved concept of power transmission conducted by the Vertol Division of Boeing under Contract DA 44-177-AMC-101(T) modification 1. The study was initiated upon receipt of the contract modification on 17 June 1964.

Phase II comprised design, fabrication, and load testing of one form of the conformal gear geometry. The gears were hardened and ground in accordance with aircraft gear technology. Capacity was determined by a programmed increase in load until failure was attained. Condition of the gears at the end of testing indicated corrective approaches for increased load-carrying capacity. Measurements of tooth bending strains were made to analyze the effects of conformal gear contact as the load zone passes across the gear face.

FOREWORD

Acknowledgment is made to the following organizations for their assistance and contributions in fabrication and inspection of the conformal gears during this program:

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CONTENTS

ILLUSTRATIONS

SYMBOLS

b	Half-length of axial contact
2b	Length of axial contact
β	Helix angle
D	Diameter
D_p	Diametral pitch
Δ	Deviation of circular arc profile from master involute curve
E	Young's modulus
$E°$	Roll angle in degrees to selected point on master involute
$\overline{E}$	Roll angle in radians
$E°_{SAP}$	Roll angle in degrees to start of active profile
F	Width of face
θ	Intermediate angle to derive a pressure angle
i	Reduction ratio
Ki	Moment correction factor
Ks	Stress concentration factor
L	Height of face in transverse plane
L_N	Height of face in normal plane
OD	Outside diameter
P	Tangential load in transverse plane (Torque/Pitch radius)

P_N	Tangential load in normal plane
ϕ	Pressure angle
ϕ_N	Pressure angle in normal plane
R	Pinion pitch radius
R_b	Radius of master base circle
R_b^1	Trial radius of master base circle
R_c	Rockwell hardness
R_E	Equivalent radius of curvature
Rm	Mean radius from outside diameter to start of active profile
Ro	Radius to outside diameter
R_{SAP}	Radius to start of active profile
R_1	Radius to center of conformal tooth profile arc
r	Radius of tooth profile
r^1	Trial radius of tooth profile
ρ_P	Radius of involute pinion tooth profile at selected point
ρ_G	Radius of involute gear tooth profile at selected point
S_b	Bending stress
S_c	Contact stress
T_N	Tooth thickness in normal plane
T_N^1	Tooth thickness (critical section) in normal plane

Thickness of pinion tooth

Thickness of gear tooth

INTRODUCTION

The need for a power transmission system to match the speed, size, and weight of the gas turbine prompted the Army's funding of Contract DA 44-177-AMC-101(T).

Initial investigations of power transmission indicated that, to obtain the minimum specific weight for the system, the engine rpm should be reduced at the rotor. This requires a high-ratio, high-efficiency reduction mechanism. With this in mind, a study of the conformal contact (Wildhaber-Novikov) gear tooth form was initiated to appraise its potential for aircraft applications. Phase I was devoted to the formulation of analytic method, a photoelastic investigation, and a parametric design study. Detail design, fabrication, and dynamic load testing to failure were performed in Phase II either to verify the analysis established in Phase I or to obtain enough data to modify it.

The conformal contact gear tooth form was first described by Ernest Wildhaber in a United States patent filed in 1923 and issued October 5, 1926. His patent contains every essential feature of the conformal contact gear being investigated today. Wildhaber describes circular arc profiles in both normal and transverse planes, and also the differences between concave and convex profiles to allow change in center distances.* Convex and concave surfaces on mating teeth create a band of contact which spreads to area contact under load. The contact runs axially along the face as the mating gears revolve (see Figure 1). By comparison to involute teeth, the conformal shape is not conjugate in the plane of rotation. For constant velocity transmission, the gear is made helical. To operate without interruption, the gear teeth must have axial overlap; that is, the face width must be wide enough to include at least one pitch, and preferably more.

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*This last modification is generally credited to M. L. Novikov, who received a USSR Patent in 1956. The chief value of Novikov's work was to revive interest in the gear form. As has frequently happened, the necessity (in this instance, a requirement for high-duty V/STOL transmissions) came long after the invention.

The particular advantage of the conformal gear is that the
two functions of gearing -- transmitting both load and
uniform angular motion -- are separate and distinct. The
motion-carrying function is entirely axial so that the tooth
can have the profile best suited to contact capacity and
strength. By contrast, the involute gear is restricted to
a conjugate geometry (see Figure 2) which prevents an
increase in contact capacity by changing tooth size (dia-
metral pitch). Increases in contact capacity must be
obtained by improved ability to withstand high stresses,
by case carburizing, for example, rather than by changing
tooth form.

Material improvements can be applied to the conformal gear
as well. The conformal gear can benefit from the hardened
surfaces and ductile core common to modern aircraft in-
volute gearing, and the extent of benefit is consistent
with involute-gear experience.

The analysis indicated that the hardened and ground con-
formal gear required a face-to-diameter ratio of 0.75 or
more to realize load-carrying advantage over the involute
gear. However, the test gears had a lesser, and entirely
practical, face-to-diameter ratio (one-half), yet their
load-carrying capacity was found comparable to involutes
of similar size and weight. It is important to note that
parity was achieved at an early stage of conformal technol-
ogy, and despite major problems. The capacity of a
developed conformal gear should therefore be considerably
greater than that indicated by the analysis.
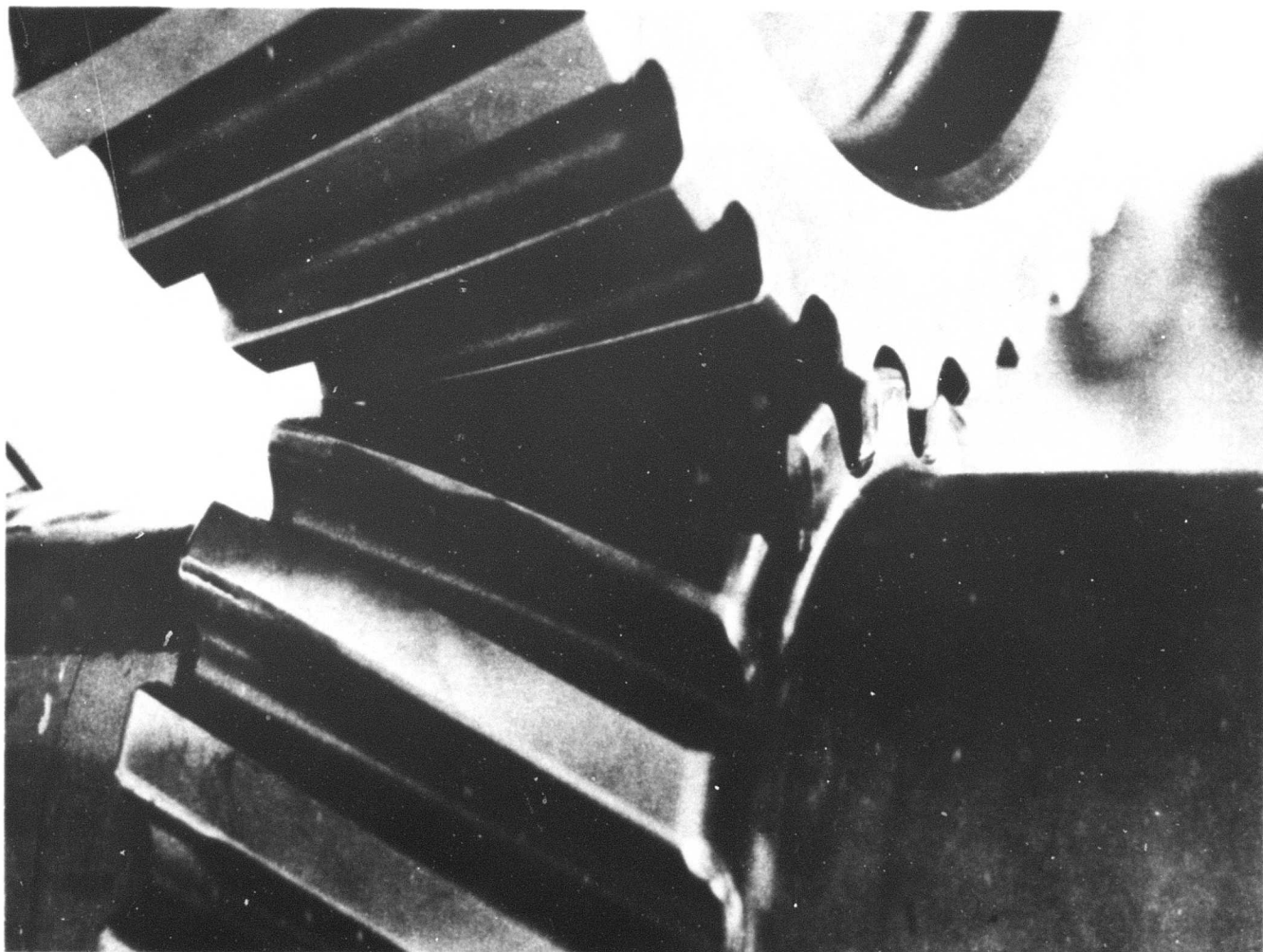
The conformal-contact gear transmits constant
angular velocity by successive contacts across
the gear face. Each tooth contacts once during
a full revolution. Sliding in the profile plane
does not theoretically occur. The mating profiles
can therefore be designed to conform and thus
reduce contact stress as compared to the involute
form.

Figure 1. Conformal Gear in Mesh
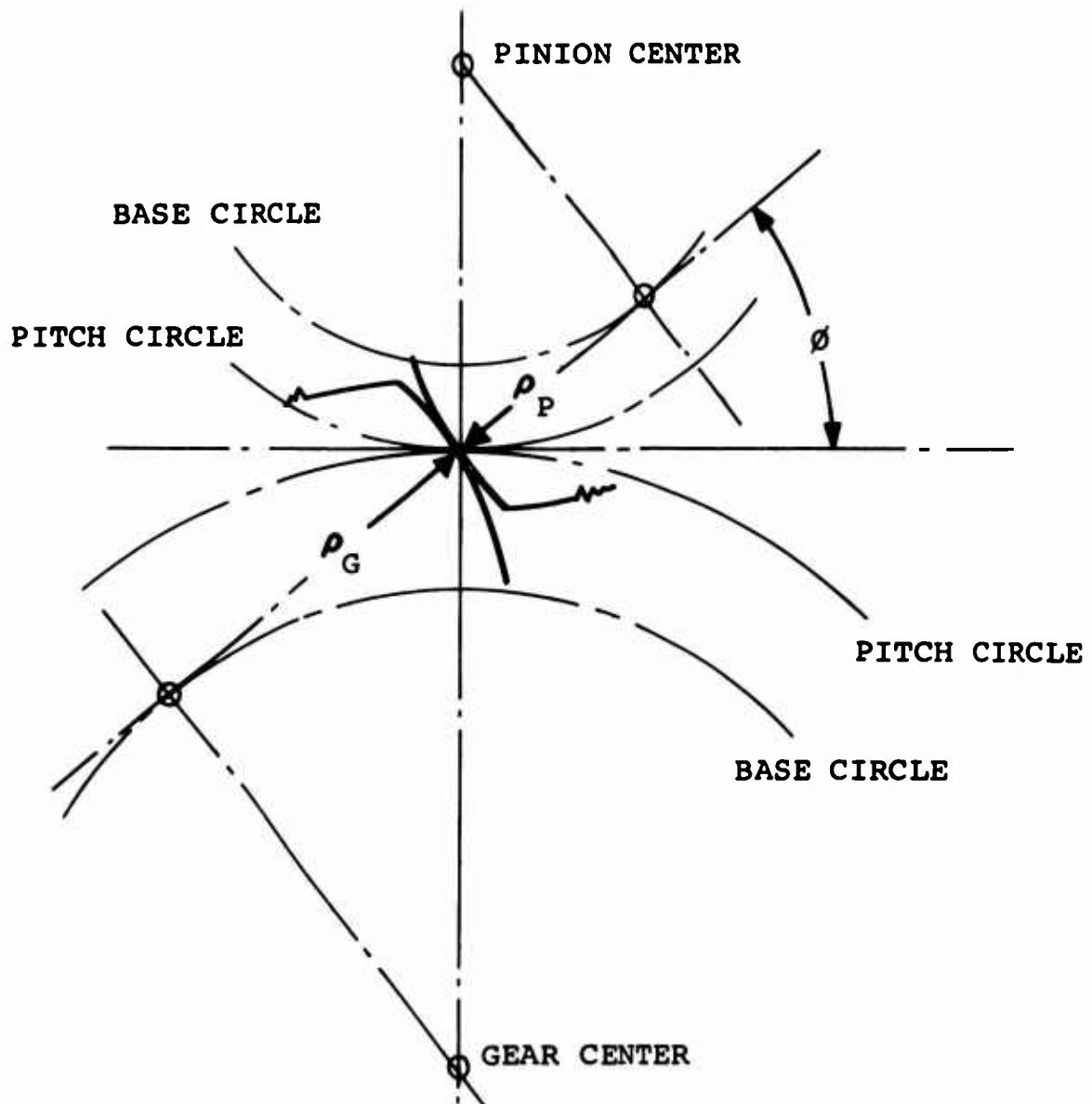
$$\text{CONTACT STRESS} = K \sqrt{\frac{\text{TOOTH LOAD}}{\text{FACE} \times \text{COS}\ \emptyset} \cdot \frac{\rho_G + \rho_P}{\rho_G \times \rho_P}}$$

● ρ_P and ρ_G are invariable for a given pitch diameter and pressure angle $\emptyset$. Therefore, contact capacity varies as $\sqrt{\text{face}}$.

● The effect of face increase is not significant within practical limits.

● Significant increase in contact capacity requires enlargement of diameters.

Figure 2. Involute Gear Contact Capacity

CONCLUSIONS

1. The conformal test gears demonstrated load-carrying ability equivalent to that expected of aircraft involute gears.

2. Load capacity can be increased by modifications in the conformal tooth to distribute load better.

3. Gearing with increased load capacity will improve upon the power-to-weight ratio predicted by the original analytical estimate.

4. Contact stresses at the failure load level correlated with the analytical method developed in Phase I.

5. Inspection techniques and equipment must be developed in parallel with the conformal tooth form.

RECOMMENDATIONS

1. Establish methods and procedures to measure and inspect more accurately the conformal tooth form along the entire face width.

2. Modify the conformal tooth form as indicated by the results of this program. Increase load capacity by eliminating tip interference and by modifying helix angle.

3. Determing the load-carrying capacity of the modified conformal gears.

ANALYSIS OF PROBLEM

The analytical method used to design the test gears was
developed in Phase I of the study. This method, like those
used in involute gear design, considers beam bending and
surface contact stresses.

As in all gear analysis, the stress values obtained are
index numbers. Allowable stresses must be empirically
determined. The first approach toward this has been made
in the testing conducted under this program.

Analysis of the conformal gear is complicated by the
allowable latitude of profile shape and profile conformity.
The analysis now considers only the circular arc profile
with full conformity between concave and convex teeth.
Nevertheless, it has proven useful for estimation of load-
carrying capacity of the test gear set. The analytical
method is described below.

BENDING STRESS

The tooth bending stress depends upon the tooth load and
upon the distribution of that load. In the profile plane,
the conformal contact system produces a line load perpen-
dicular to the axis when the radii match, and a point load
when the radii differ. In the axial plane, the curves of
the helixes oppose and produce a point loading. Therefore,
when elastic deformation is neglected, the area of loading
is normally a point.

Using the Hertz equations for bodies in contact, the area
of loading can be determined for a practical case where
deformation exists. The two radii of the contact body are
calculated, and then the equivalent radius of a curved body
in contact with a plane is determined. The equivalent
radius depends on the helix angle, the ratio, the size of
gear, and the height of the tooth (see Figure 3). From
the equivalent radius, the band width of contact axially
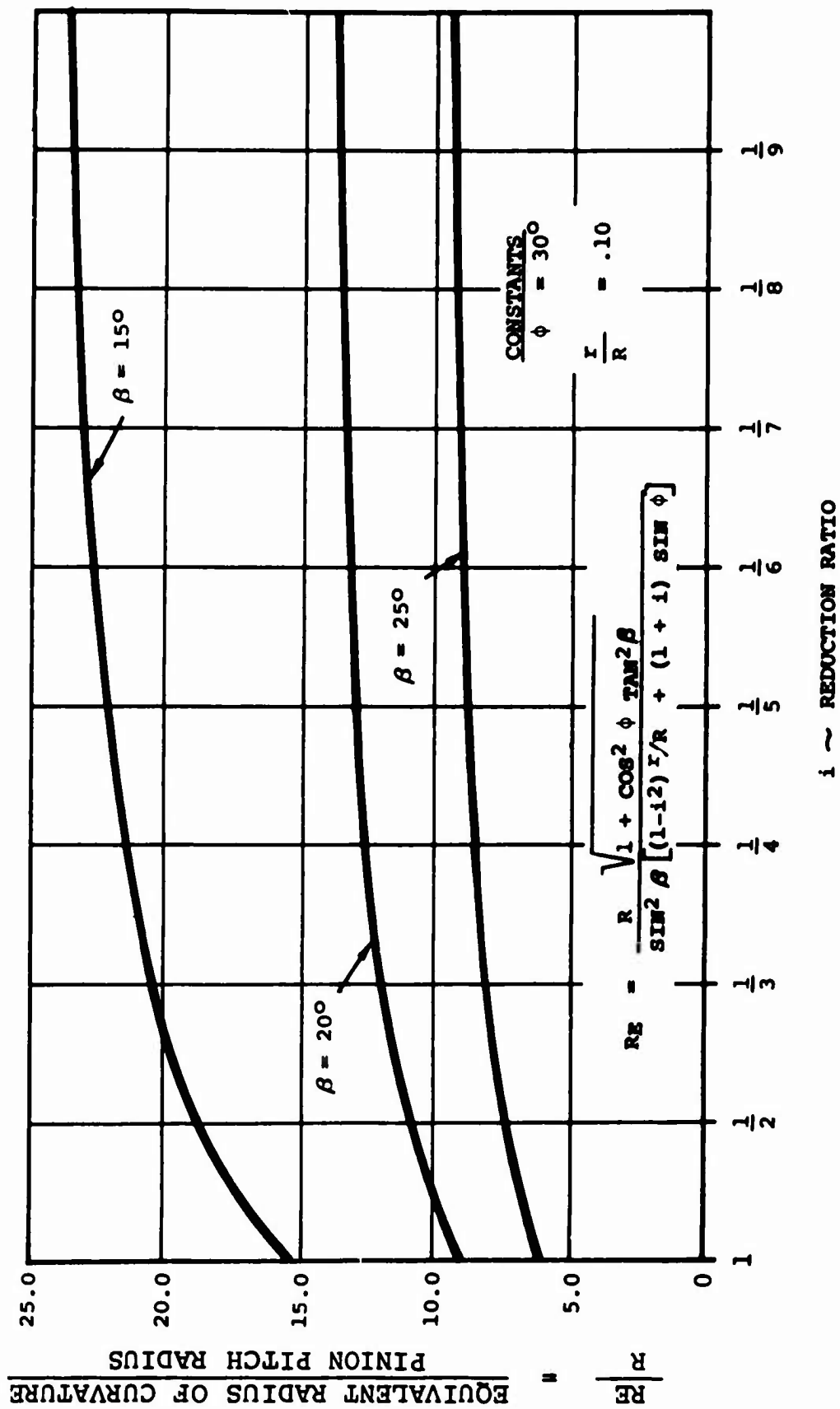along the tooth face is obtained.

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Figure 3. Equivalent Radius vs Gear Ratio

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To relate area of loading to stress at the tooth root, the Wellauer-Seireg paper* was applied. This paper deals with the stress distribution at the base of a cantilever plate. A plate is differentiated in this case from a beam, in that the load is not uniform along its length (spanwise) and in that its aspect ratio is 4 to 1 or more. A concentrated load produces a moment distribution curve of a specific shape along the fixed edge. This shape has been determined analytically and verified experimentally.

With the conformal gear, the problem is to determine the moment intensity under load patterns which extend for various spanwise distances, and in which the load distribution is assumed to be elliptical. The load span is dependent upon the mating radii of the gear and also upon the load deformation. A given gear set will enlarge its loading band as the tangential load is increased. The effect is a nonproportional increase in root bending, nonproportional because as the load is increased it diffuses further into the hitherto unstressed root area.

A correction factor for bending stress (Ki) was obtained and plotted (Figure 4). It can be seen that as contact band-to-total-face-width ratio increases, the correction factor and the bending stress decrease. From a concentrated load to a full-width load, the expected change in bending stress is 3 to 1. However, the majority of study examples had load-width-to-total-width ratios of about one quarter.

The bending stress equation also includes Ks, the stress concentration factor. A conventional equation which accounts for various beam proportions and root radii was used initially. The photoelastic technique was used with varying root shapes to further determine Ks. The results of the photoelastic method agree generally with the analytical factor.

To complete the bending stress determination, the radial compressive stress was calculated. This is caused by the tangential tooth load's acting upon the inclined tooth face. The effective area was assumed as the band width having the same elliptical distribution as previously. Acting as a

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* E. J. Wellauer and A. Seireg, "Bending Strength of Gear Teeth by Cantilever-Plate Theory", ASME Paper No. 59-A-50.

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$$\frac{\text{MAXIMUM MOMENT}}{\text{AVERAGE MOMENT}} = (K_1) \text{ VS \% CONTACT } \left(\frac{2b}{F}\right)$$

$2b/F$ (% CONTACT)
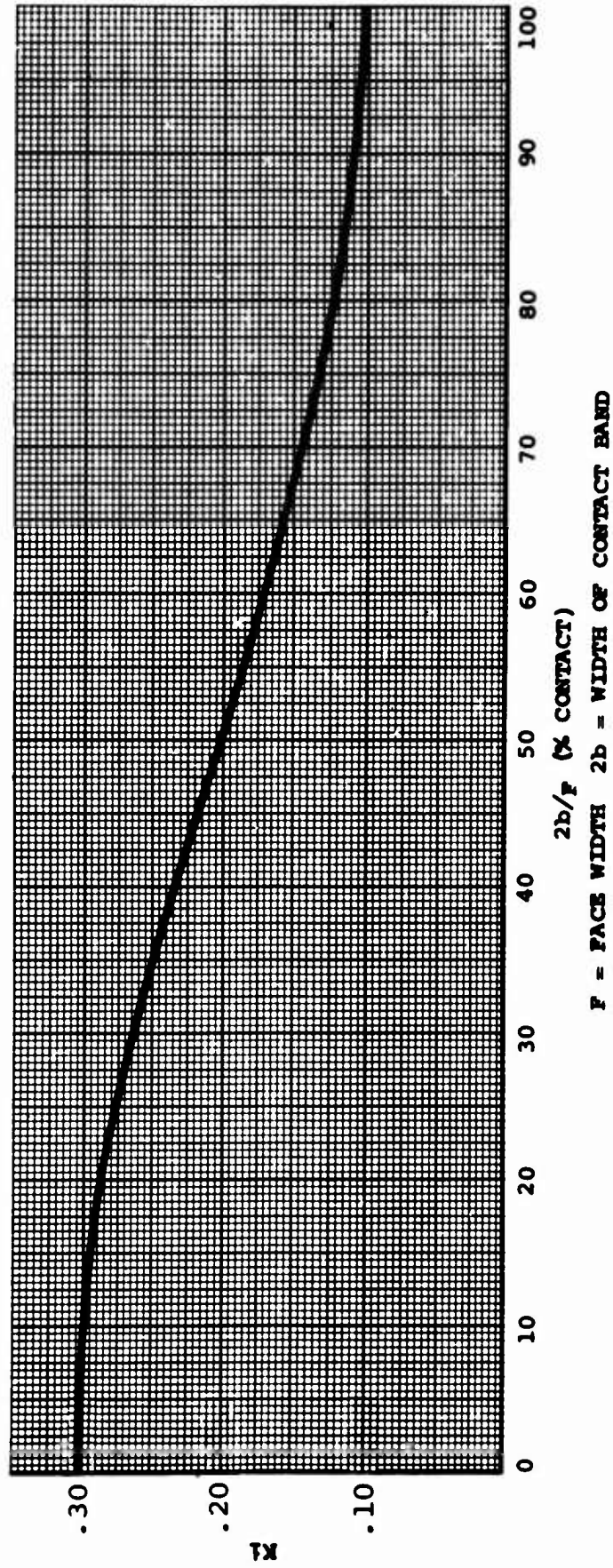
F = FACE WIDTH   2b = WIDTH OF CONTACT BAND

Figure 4.   Moment Factor vs Contact Ratio

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bending moment relief, this radial component adds to the stress on the compression side of the beam and subtracts from the tension side. Its effect is therefore beneficial, since tooth fatigue failure occurs on the tension side because of the characteristic lower endurance limit.

CONTACT STRESS

The contact stress assumes the same elliptical load distribution over the band of contact. The other axis of the contact area is assumed as the normal height of the tooth profile. This full conformity will not actually pertain, particularly at partial load, but less than full conformity will increase the elastic deformation in the spanwise (axial) direction and increase the length of the band of contact, so the assumption is realistic enough to be useful.

The influence of the tooth geometric variables upon the bending and contact stress levels was considered. It was apparent that the relationship between bending and contact stresses depends upon tooth thickness for beam strength and upon tooth profile radius and helix angle for area of contact. The balance of tooth bending and contact stresses is an important step in optimizing the gear. Detail studies in which one variable was changed to determine the stresses showed that beam strength was the limiting factor. The conformal gear's contact capacity was believed to be superior to its bending capacity, unless the gear tooth is thickened to extremes by normal involute standards.

The following equations were used in this study to obtain bending and contact stresses:

To Find Normal Load:

$$P_N = \frac{P\sqrt{1 + \cos^2\phi \, \tan^2\beta}}{\cos\phi}$$

Where

P_N is load normal to face

P is tangential load $= \dfrac{Torque}{Pitch\ Radius}$

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ϕ is pressure angle

β is helix angle

To Find Height of Contact Band:

$$L = 2 \sin \phi \ r$$

$$L_N = L \frac{\sin \phi}{\sin \phi_N}$$

$$\tan \phi_N = \tan \phi \ \cos \beta$$

Where L is height of face in transverse plane

L_N is height of face in normal plane

ϕ_N is normal pressure angle

r is radius of tooth profile

To Find Length of Axial Contact Band:

$$2b = 2.15 \sqrt{\frac{2 \ P_N \ R_E}{E \ L_N}}$$

Where 2b is length of band

R_E is equivalent radius (see Figure 3)

E is Young's modulus

To Find Bending Stress:

$$S_b = \frac{6 \ K_i \ K_c \ P_N \ \cos \phi_N}{(T^1_N)^2} - \frac{P_N \ \sin \phi_N}{2T^1_N \ b}$$

Where S_b is bending stress at tension fillet

K_i is correction factor (see Figure 3)

K_s is concentration factor

T^1_N is critical section in normal plane

b is half length of axial contact band

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To Find Contact Stress: ·

$$S_c = \frac{4}{\pi} \frac{P_N}{2b \quad L_N}$$

To Determine Diametral Pitch:

$$D_p = \frac{\text{overlap} \times \pi}{\text{Face} \times \tan \beta}$$

To Determine Pinion Thickness:

$$T_1 = \left(\frac{T_1}{T_2} \times \frac{\pi}{D_p} \right) - \text{Backlash}$$

Where

T_1 is   pinion tooth thickness

T_2 is   gear tooth thickness

D_p is   diametral pitch

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DESCRIPTION OF TEST ITEM

DESIGN

The four test gears used in this program were manufactured
by the Steel Products Engineering Company, Division of
Kelsey-Hayes, Springfield Ohio, and finish ground by the
National Broach and Machine Company, Detroit, Michigan.

The SK13264 conformal test gears were made to the following
design specifications (for additional dimensional data, see
Figure 5):

	Gear Set Number 1		Number 2	
	SK13264-1	-2	-3	-4
1. Pressure Angle	30°		30°	
2. Diametral Pitch	4.50		4.50	
3. Number of Teeth	16	38	16	38
4. Pitch Diameter	3.555	8.444	3.555	8.444
5. Face Width	1.950		2.132	
6. F/D Ratio	0.50		0.55	
7. Helix Angle	25° 13'		25° 13'	
8. Axial Overlap	1.20		1.40	

The two gear sets are identical except that face width was
varied to investigate axial overlap, an important design
parameter. This variation did not require changes in tooling
or manufacturing technique. The design specifications
selected for the test gears reflect the assumptions and
methods of Phase I. The specifications were chosen to pro-
vide equal bending strength and surface contact capacity.

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These are at least four important geometric variables that
determine strength and contact capacity within a given
gear blank size: tooth thickness, overlap ratio, profile
radius, and pressure angle. Subsidiary to these, but also
important, is the profile mismatch, which is selected
from the expected change in operating center distance. By
proper portioning of the variables, the necessary balance
of strength and contact capacity may be obtained.

A discussion of the test gear design considerations follows:

Tooth Thickness

Tooth thickness depends upon helix angle and face width-to-
diameter ratio (Figure 6). Face width-to-diameter ratio
was made 0.5, on the assumption that this represents a
condition where deflection and support problems are minimal.

The analytical method indicated that bending strength
should be maximized to balance contact capacity. The helix
angle was therefore set at a practical high limit of 25 degrees.
This produced a thick tooth, with maximum section modulus
at the base. The choice of helix angle also influences
the contact capacity. Increasing the helix decreases the
curvature radii in the axial direction and thus the axial
length of the band of contact is decreased. Contact stress
is thereby increased. The choice of helix angle is a
powerful stress balancing factor, since it has a reverse
effect on the two stress parameters which determine load
capacity.

Results of the testing indicate that the design was unduly
prejudiced in favor of strength. An immediate reaction would
be to reduce helix angle to increase contact capacity.

Overlap Ratio

Overlap ratio is the face width divided by the axial pitch.
Overlap ratios of 1.2, or more, are recommended to equalize
bending stress between middle of tooth and end of tooth.
More overlap than is necessary to accomplish this increases
face width with no increase in load capacity. Less overlap
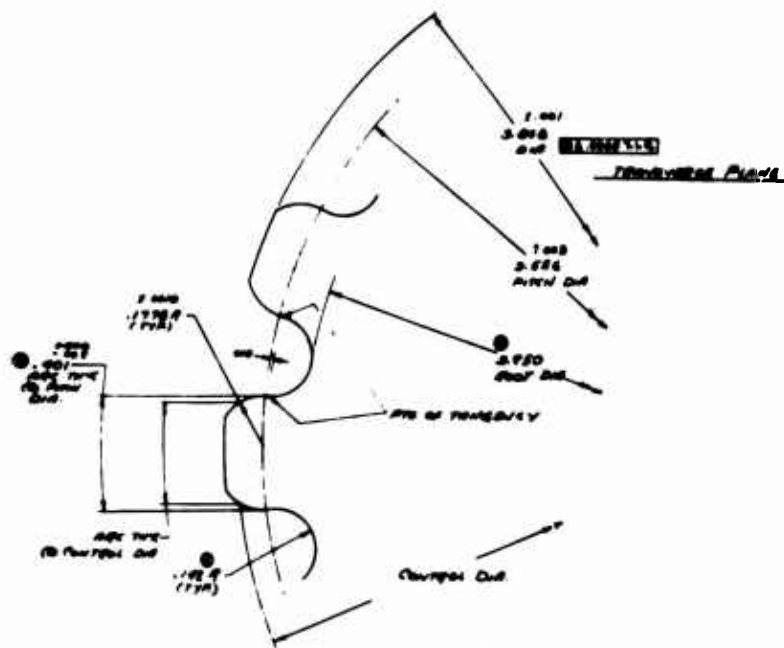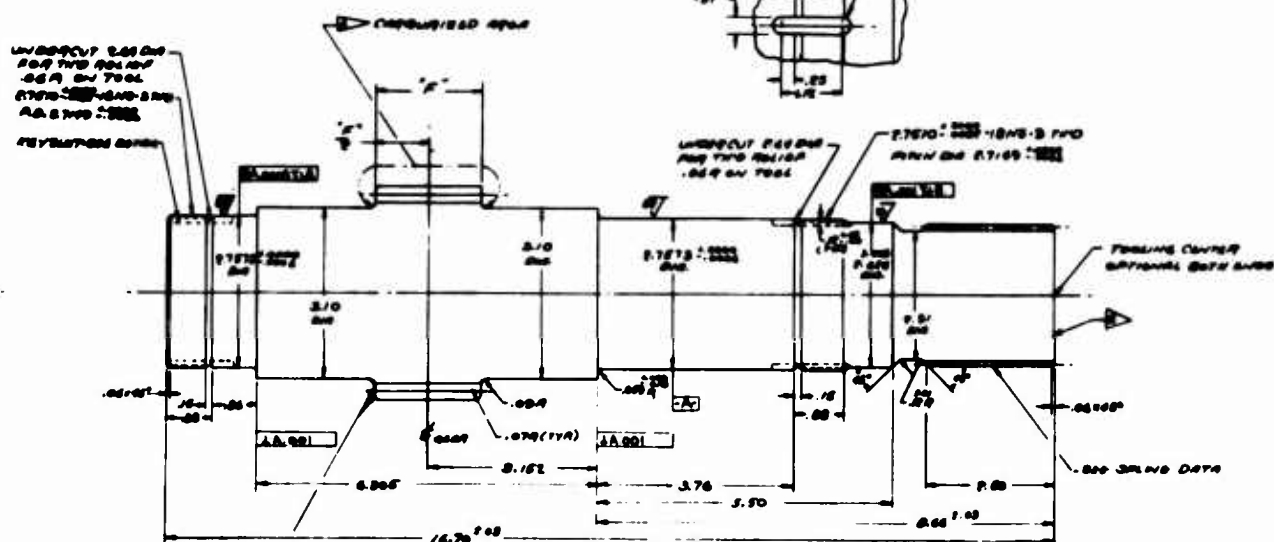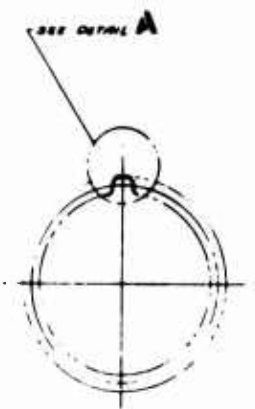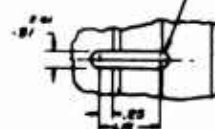may result in premature bending failure. It is therefore

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Figure 5.   Drawing SK13264

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Figure 6. Tooth Thickness vs Helix Angle

desirable to develop an understanding of overlap ratio effect. To this end, the test sets were made in two overlap ratios. Because of other overriding differences, the effect of varying overlap ratio was not observed. However, the strain gage survey undertaken after the load tests provides an insight into the load distribution across the tooth, which will be useful in future designs.

Profile Radius

A profile radius of curvature of 10 percent of the pitch radius was a practical maximum to obtain an adequate top land width with the 30-degree pressure angle. The selection of a 10-percent mismatch of convex and concave profile radii was based upon the expected accuracies of aircraft involute manufacturing experience, and the recorded accuracies of center distance and alignment of the test stand.

Pressure Angle

The pressure angle of 30 degrees was selected for maximum bending strength. A high angle gives a theoretical load line with minimum moment arm to the critical section, plus a significant compressive relief component.

FABRICATION

The manufacturing procedure and process sequence for the conformal contact test gears were as follows:

1. Forge individual biscuit of 9310 vacuum melt steel

2. Rough-machine gear blank and stress relieve

3. Final-machine gear blank

4. Hob (cut) gear teeth

5. Carburize gear teeth

6. Heat treat (harden and draw)

7. Grind faces and journals

8. Grind teeth

9. Make final inspection

10. Shot peen teeth

This sequence is normal for Vertol Division production gears. Standard gear production machine tools were used although special development effort in hob design (see Figure 7), grinding, and inspection was required to produce the conformal tooth.

Checks by optical projection and comparison with the template masters revealed a non-uniform grinding stock allowance on both members. To ensure material cleanup and to equalize stock removal, grinding stock removal was increased 0.005 inch each side of the pinion and decreased 0.008 inch each side of the gear. This reduced the calculated minimum assembled backlash from 0.012 inch to 0.006. In a small test quantity of gears it becomes necessary to increase the grinding stock allowance to compensate for unpredictable distortions during heat treatment. Consequently, more of the carburized compressive layer is removed, and removed less uniformly than in production-quantity gears. Therefore, to compensate for this condition, the two completed sets of conformal test gears were shot-peened at a Vertol-approved facility to improve residual compressive stress.

A review was made to determine the most feasible method for finish grinding the conformal contact gear. Two types of grinding were compared: generated grinding and form grinding. The results indicated that, while either method is suited to the conformal concept, form grinding would be more suitable for the test gears because of the diametral pitch selected. Before finish grinding of the test gears, the form grinding method was evaluated by finish grinding dummy gear blanks. Nital etch inspection of the first ground pinion from the dummy set (verified by subsequent nital etch and magnaglo inspection at Vertol Division) revealed heavy burns at approximately the pitch diameter. The difficulty can be attributed to the profile and large full fillets of the convex tooth and to the resulting
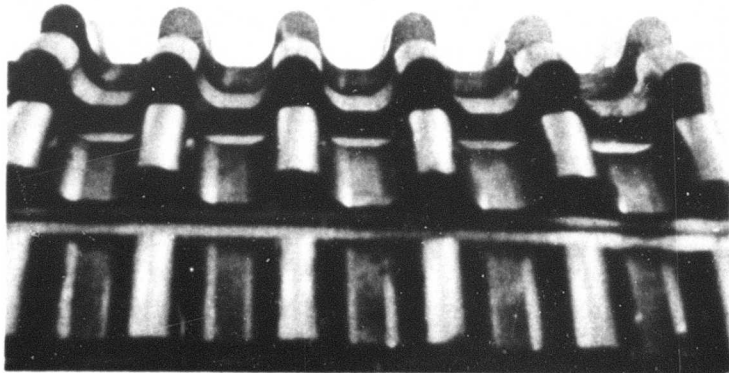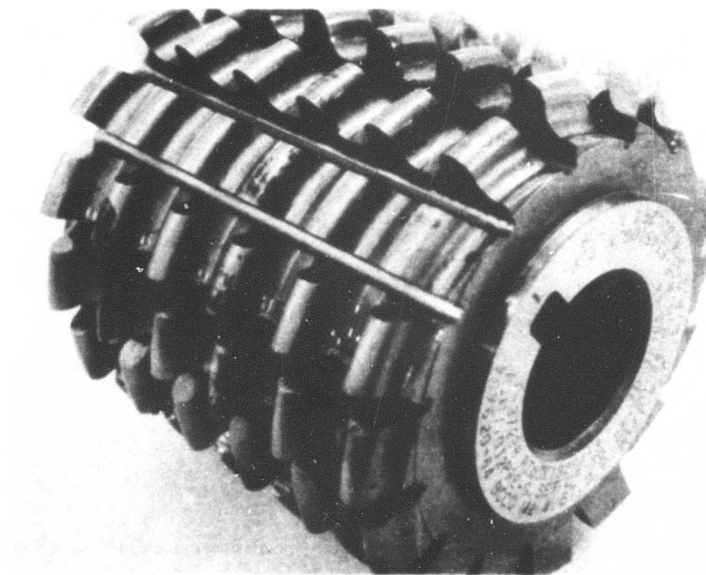
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Figure 7.   Conformal Pinion Hob

transition portion near the pitch diameter. The problem
was overcome by modification of the grinding technique.
Grinding damage was not noted on the test gears.

INSPECTION

A study was conducted to determine the most practical
method for checking tooth form (including profile) and
fillet contour. It was decided to use optical projection
by surface illumination on a 30-inch screen optical
projector. Charts of the tooth profiles, before and after
finish grinding, were made at a magnified scale (20X) for
comparison with the projected image (see Figure 8). This
method has inherent limitations for direct observation of
the conformal gear: helical curvature limits inspection
of the tooth profile to the ends of the gear face, and
edge breaking of the gear teeth distorts the projected
image. An indirect method was employed in this program
to eliminate the second problem. A shim 0.002 inch
thick was fastened to the end of the gear blank, hobbed
and ground with the gear, and then removed and projected
for comparison with the charts. The probable inaccuracy
of profile measurement by the method used is believed to
approximate 0.0005 at the gear. This estimate combines
the chart tolerance and the limits of visual discernment.

There is an obvious need to develop an inspection technique
that can define the tooth profile at any transverse plane
with accuracy comparable to involute gear requirements
of within 0.0001 inch. Several such techniques are under
consideration. One method, an adaptation of one now used
by Automation Gages, Inc., to inspect complex shapes,
employs a coordinate tracer and optical projection (Figure
9). It appears to be able to map conformal tooth profiles
of any type of curvature, concave or convex. The effort
required to adapt the apparatus and techniques has not yet
been defined. A developed refinement of the system
sketched is the use of photocell pickups to increase
accuracy beyond that limitation of the eye. The output of
the photocell scanner may be displayed by a digital encoder.
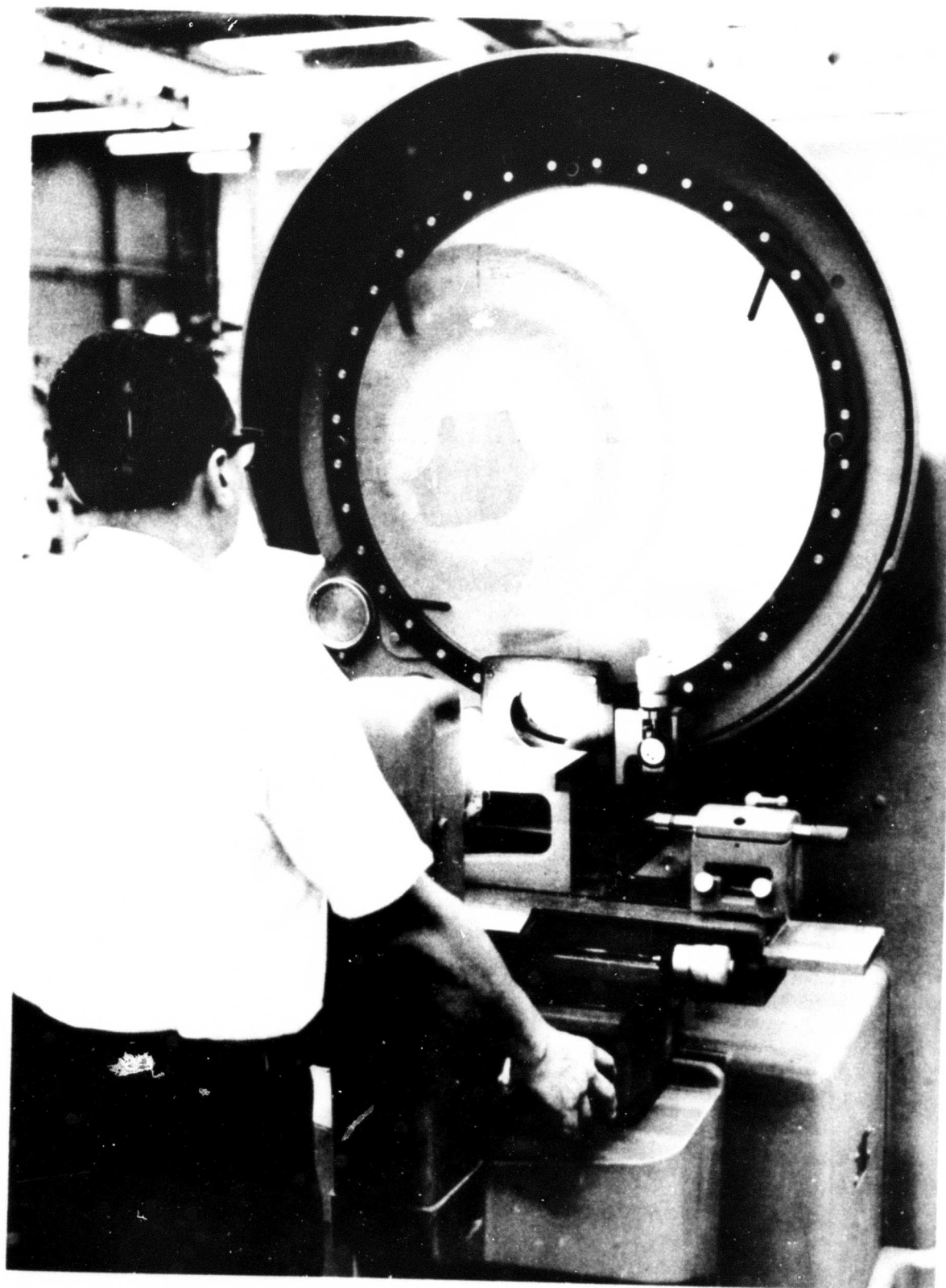
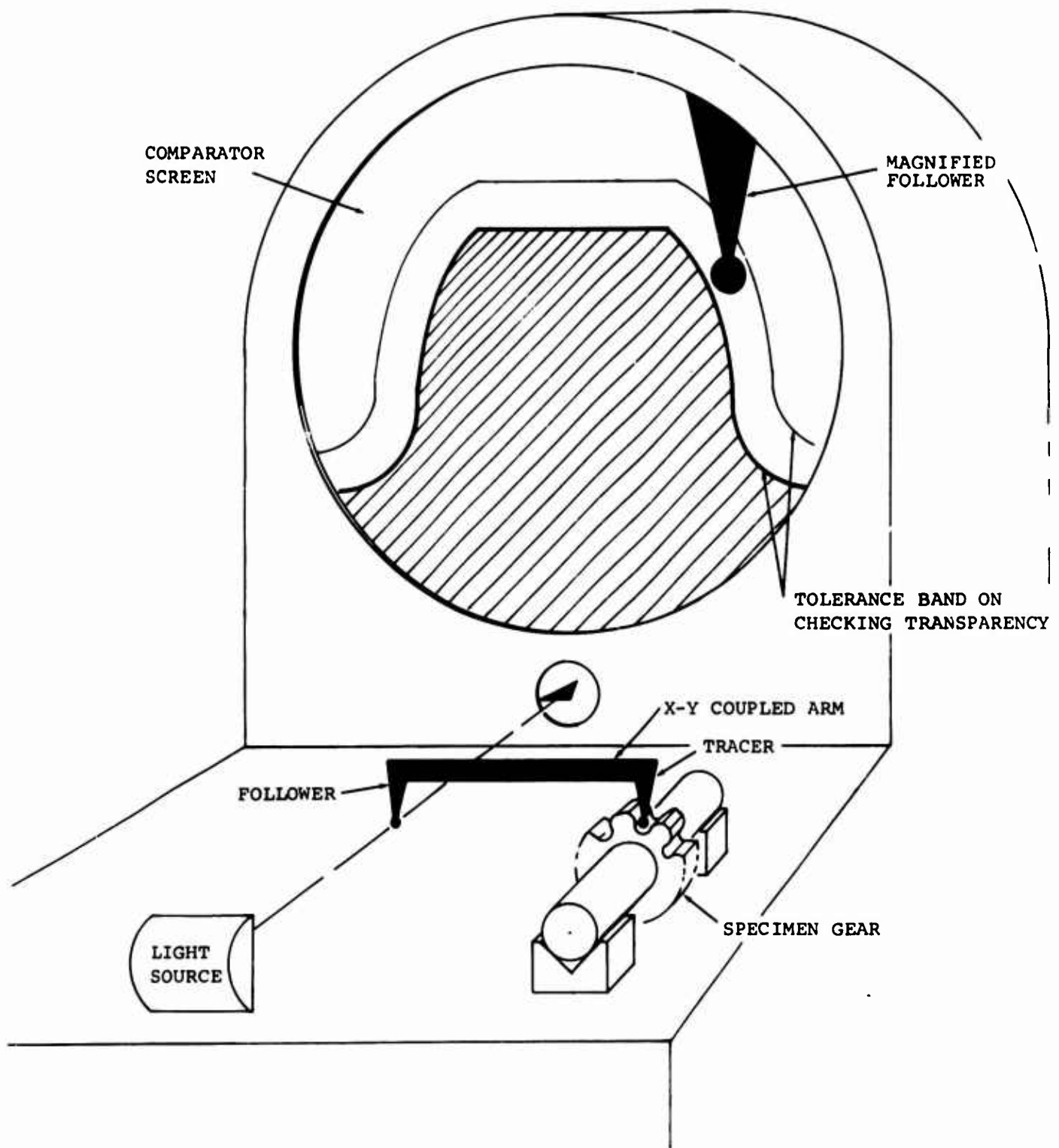Figure 8. Comparator and Chart

Figure 9. Coordinate Tracer Schematic

A more immediate method, which may be used with existing inspection equipment, adapts an involute checker to the circular-arc profile. At the moment, it appears limited to the convex tooth. The involute checking equipment used industry-wide traces the tooth profile on a strip chart as the gear is rotated. A perfect involute appears as a straight line; departures from the involute cause deviations from the straight line, highly magnified and directly measurable on the chart paper. The technique can thus be adapted to circular-arc profiles, provided only that the reference involute fits the circular arc well enough that the maximum deviation is within the machine range. The reference involute and the offsets are mathematically obtainable (refer to the Appendix). The base circle from which the reference involute is constructed can be readily set into the involute checker, and a circular-arc profile can then be measured. The resulting chart trace is shown in the Appendix. A basic check of machine accuracy is obtainable by the Michigan Tool Company Roll-Pin device; it is widely used as an involute machine checking standard. With proper dimensional selection, it produces a chart identical to that obtained from a convex circular-arc gear profile.

Individual gear tooth element tolerances, such as lead error and tooth-to-tooth spacing error, were measured by conventional inspection machines and methods.

To complement the detail inspection, light load pattern checks and noise level tests (gear speeder) were conducted on the dummy pinion and gear at varied center distances. At the nominal 6.000-inch center distance, the pattern on the gear member was an axial line just beneath the tip radius. The pattern on the pinion member was an axial line just above the pitch diameter. At 5.994-inch center distance, the pattern shifted down on the gear member and up on the pinion member. The noise level was higher than the previous check. At 6.006-inch center distance, the patterns shifted back to high on the gear and low on the pinion, with a more pronounced line evidencing increased tip radius loading. The noise level was comparable to or better than that obtained at the 6.000-inch center distance (see Figure 10).
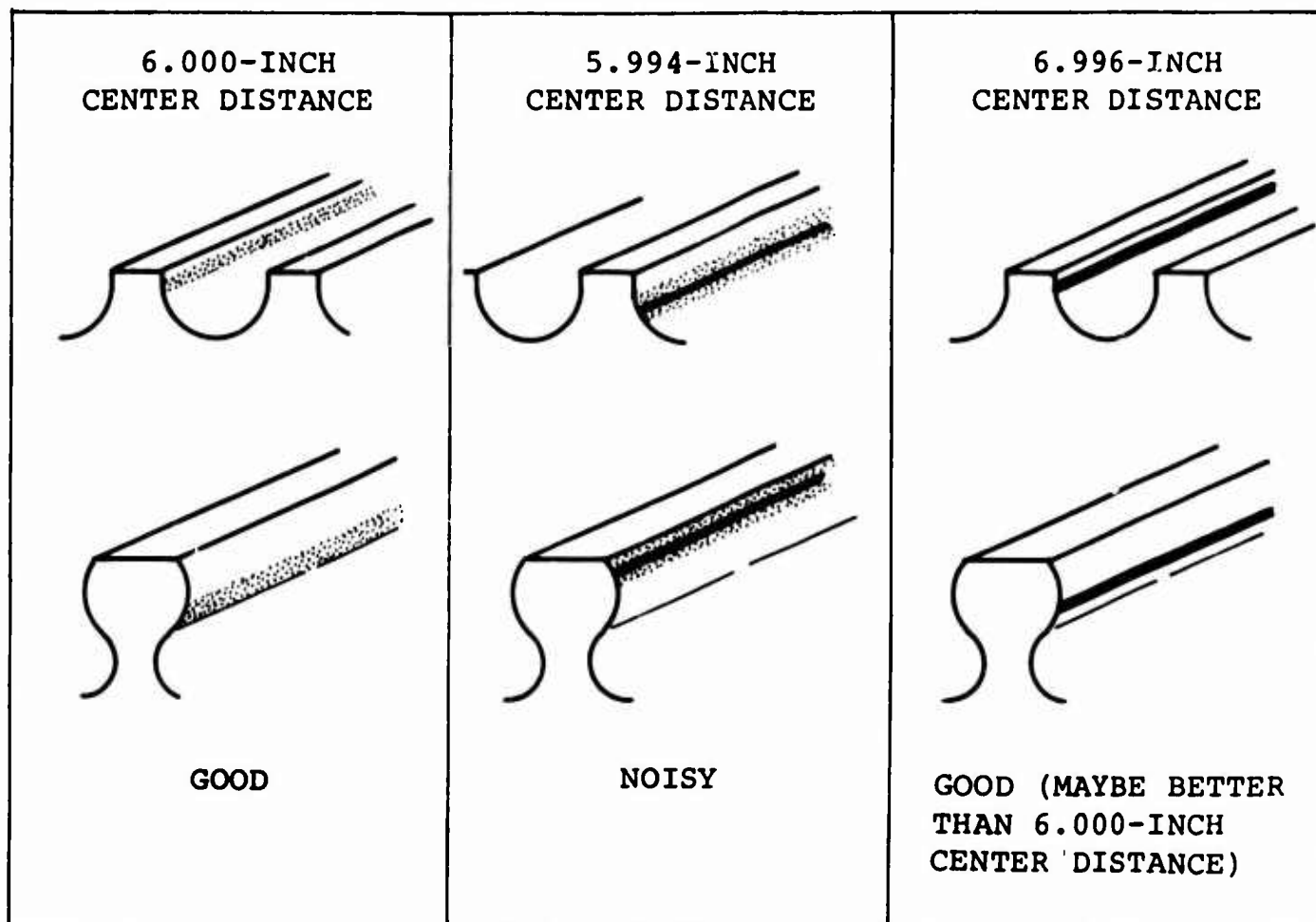
Figure 10. Patterns of Dummy Pinion (Top Row) and
Gear (Bottom Row) at Various Center Distances

LOAD TESTS

EQUIPMENT

Testing was conducted in a Vertol-owned regenerative (four square) load test stand built specifically for gear research (see Figures 11 and 12). The two gear cases are designed to be rigid and stable under all loading conditions. Housings were through-bored for maximum accuracy. Torque, rpm, and lubrication can be varied over a wide range.

Accelerometers were attached to both the slave and test gearboxes and connected to a dual-channel oscilloscope. Changes in vibration signatures for both gear cases can be observed simultaneously and used as an indication of impending gear failure.

Each gearbox lubrication system includes an individual reservoir, pump, control, indicators, filler, and jets. All controls and indicators are mounted in the console. Lubrication oil volume is controlled by a bypass valve. Temperature is indicated both in and out of the test gearbox. Pressure is indicated at a point just upstream of the jets.

Atlantic Refining Company Premier 12 gear lubricant was used throughout the test program; it contains extreme pressure additives in the form of lead soaps. Vertol Division has used Premier 12 in previous gear research testing*; the results of the previous testing were used as a base line for evaluating the conformal gear.

Lubrication was provided by oil jet on the out-of-mesh side. The flow rate, approximately 2.0 gallons per minute for gear set number 1, was increased to 2.9 gallons per minute before testing gear set number 2. The added flow was directed to the entering side of mesh, where surface deterioration had been observed during the tests of gear set number 1. Lubricant pressure, temperature, and viscosity were controlled to the following values during the

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*J. Mack and B. D'Angelo, "Evaluation of the Effect of Gear Tooth Grinding Methods on Fatigue Life of Spur Gears," Technical Report R-342A, The Vertol Division of Boeing, August 1964.
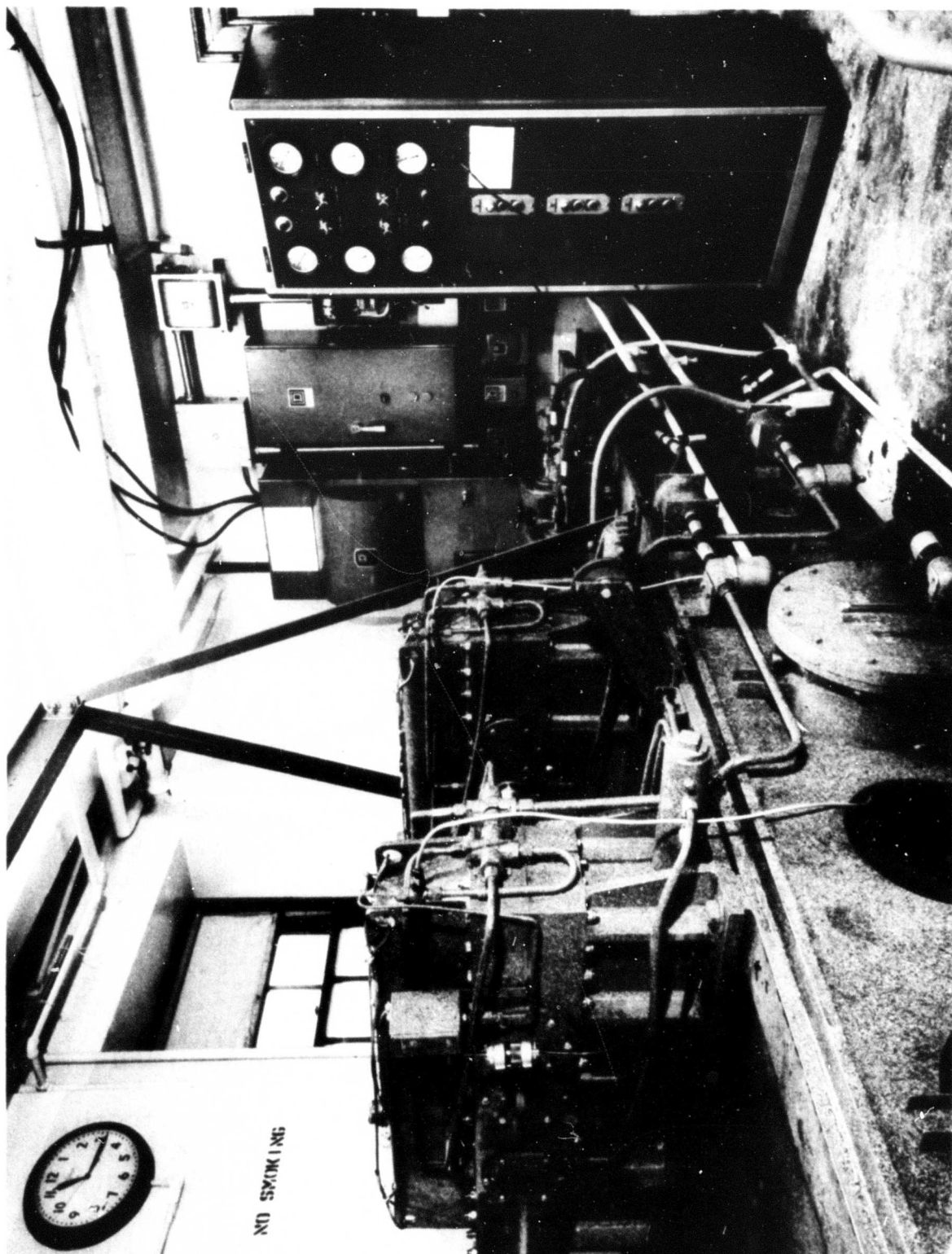
Figure 11. Gear Research Test Stand

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tests:

1. Operating Temperature: 125 ± 15°F

2. Operating Pressure: 60 ± 15 PSI

3. Viscosity at Operating Temperature: 300 SSU

VARIABLES

The primary test variables were shaft torque and test
cycles (time). Gear load is a function of shaft torque.

Torque

Torque was applied with a lever system at the beginning of
each test run. Torque levels were observed on an SR-4
instrument. The procedure for this test program was to
check torque after 30 minutes of running, then to
recheck every 2 hours during the test. Deviation from
the initial target torque was held to ± 5 percent.

The torquemeter shaft was calibrated before and after the
test series on a Riehle deadweight torsion test machine.
The same instrument was used during the calibration and
testing. Recalibration curves agreed with the initial
curve within 2 percent.

Test Cycles

Test cycles were determined by a log record of running time
and an elapsed time meter in the test stand console. An
electric motor driving the input shaft through a toothed
belt maintained pinion speed at 3600 rpm (346,000 tooth
loading cycles per hour).

PROCEDURE

At the initial installation of both gear sets, static tooth
patterns were recorded at various load levels, and the gear
sets were then run in for 1.73 million cycles (8 hours) at
approximately 1260 inch-pounds torque. Gear set number 1

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Figure 12.   Test Arrangement

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ST0164-11 COUPLING

MOTOR

DRIVEN SET

DRIVER TST

3600 RPM

ROTATION

TORQUE

SK/3918-1 PINION

ROTATION

1530 RPM

ST0154-7 SHAFT

SK/3918-7 GEAR

ST0164-3 COUPLING

SLAVE BOX (SPUR GEARS)

TEST STAND ST0150
CONFIGURATION
FOR CONFORMAL GEAR TESTS

SLAVE BOX
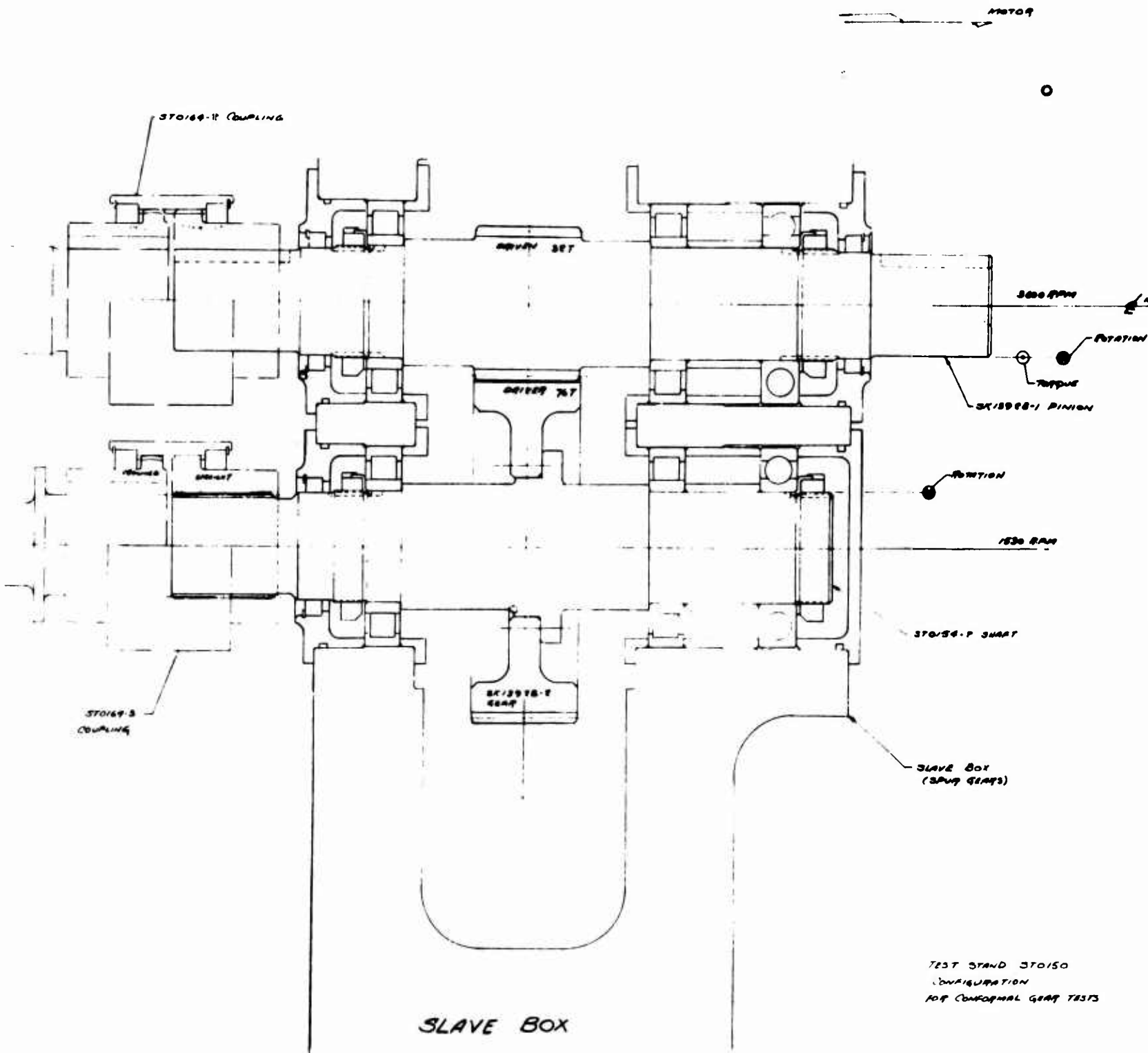
B

was run in for an additional 432,000 cycles (2 hours) after
initial pitting from run 2 had been removed by stoning.

Each gear set was given three test runs at various load
levels. For each load level, the test run was concluded
at 3.46 million cycles of load on the pinion tooth. Previ-
ous experience with involute spur gears locates the knee
of the S-N curve at between two and three million cycles.

The load test program is summarized in Table I. Stress
levels associated with test load levels are shown in
Figure 13.

RESULTS

Static Pattern Check and Run-in of Gear Set Number 1

Static pattern checks at the various load levels indicated
a tapered tooth pattern, larger at the entering side of
mesh and smaller at the leaving side (see Figure 14). The
pinion member had the deepest extension of the pattern
approximately parallel to the outside diameter, with the
taper at the top. The gear member had the opposite condi-
tion. This tapered pattern became more pronounced with
increased load, and it persisted throughout the dynamic
testing.

Visual inspection after the initial run-in period did
not reveal any adverse conditions.

Load Test Runs of Gear Set Number 1

Test run 1 showed no surface indications.

Test run 2 was completed at 133 percent of design load.
Visual inspection at this time revealed conditions similar
to gear set number 1 at the completion of test run number
2. See Figure 14, Sheet 1 for this condition.
The single pit was not as deep or pronounced as the number
1 gear set, and it originated at the very edge of both mem-
bers. It should be noted that this gear set had an increased
contact ratio (1.4), increased face width over pitch diameter
ratio, and increased oil supply to the entering side of mesh.
The pitting condition was removed by hand stoning.

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TABLE I
LOAD TEST PROGRAM

| Test Run	Stress Cycles (millions)		Pinion Torque (in.-lb)	End Condition
	Pinion	Gear		
Set No. 1	SK13264-1, Ser. P1009	SK13264-2, Ser. P1001		
Run-in	1.728	0.727	1,260	Tapered pattern
Run 1	3.460	1.450	5,700	Tapered pattern
Run 2	3.460	1.450	8,600	Pitting on both members
-	-	-	-	Pits stoned out before run-in
Run-in	0.432	0.182	1,260	Inconsistent pattern
Run 3	2.980	1.260	13,400	Surface failure of both members
Set No. 2	SK13264-3, Ser. P1005	SK13264-4, Ser. P1003		
Run-in	1.728	0.727	1,370	Tapered pattern, but to a lesser degree
Run 1	3.460	1.450	8,600	Initial signs of pitting on entrance side
-	-	-	-	Pits stoned out before run 2
Run 2	3.460	1.450	13,900	Pitting on both members on entrance side
Run 3	1.080	0.470	16,100	Surface failure of both members on entrance side
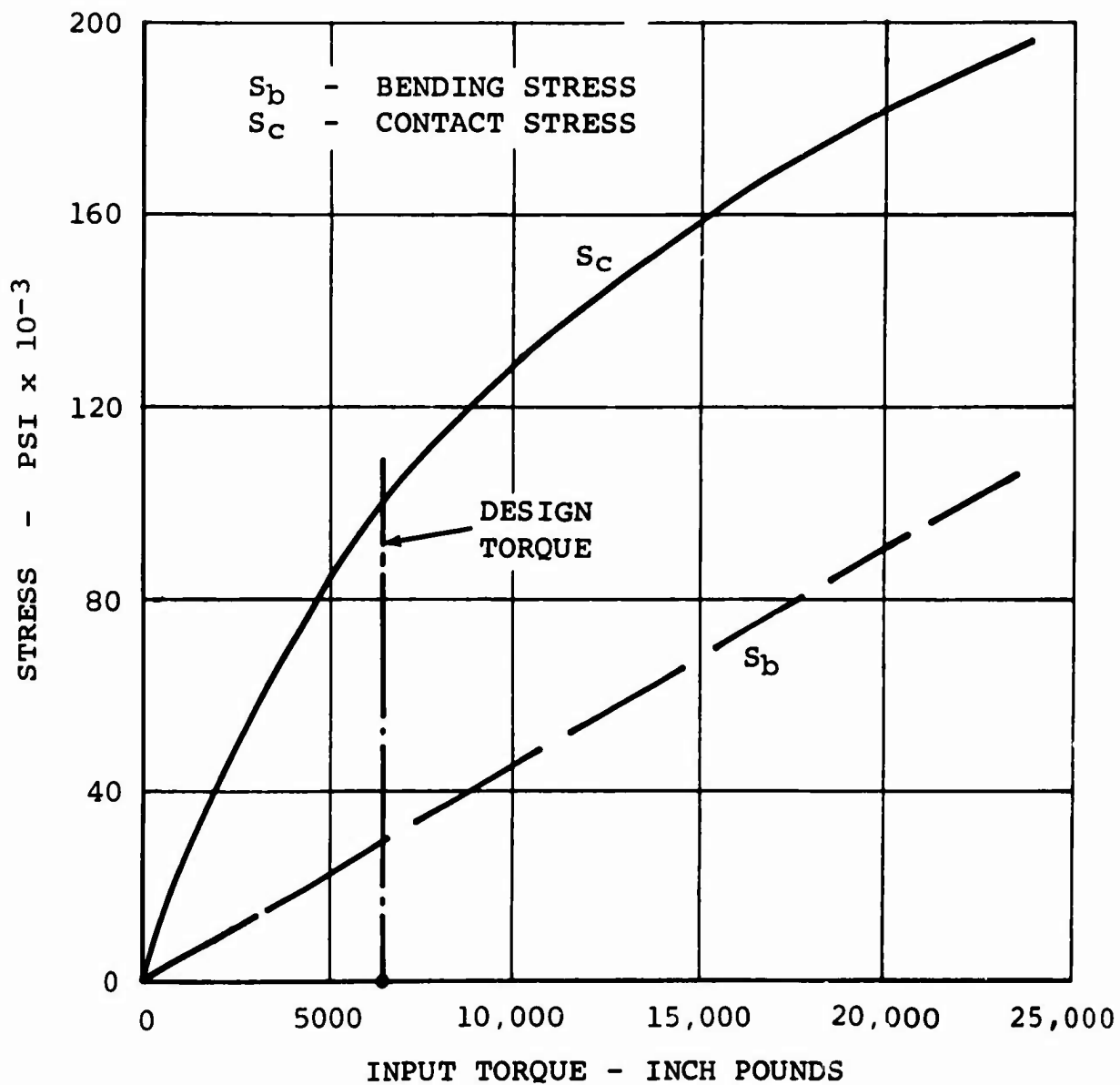
Figure 13.   Conformal Gear Stress Levels (Not
Corrected for Actual L_N)

GEAR PATTERNS AFTER RUN-IN, GEAR SET NUMBER 1

PINION SK13264-1, SER. P1009
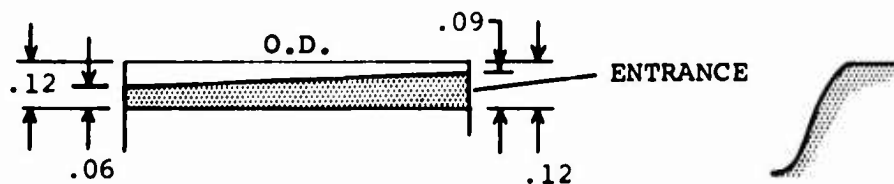
GEAR SK13264-2, SER. P100_
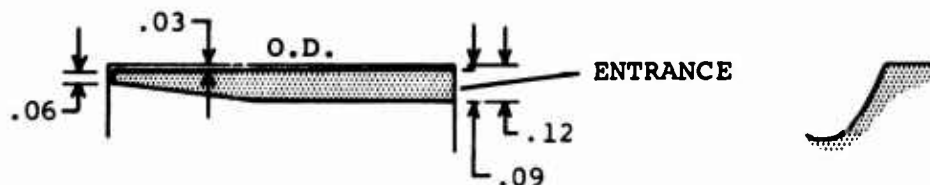
GEAR PATTERNS AFTER RUN 1, GEAR SET NUMBER 1

PINION SK13264-1, SER. P1009
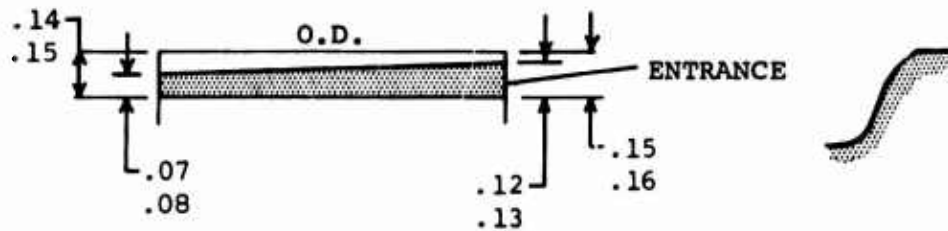
GEAR SK13264-2, SER. P1001

Figure 14.   Tooth Patterns During Load Test (Sheet 1)

PINION SK13264-1, SER. P1009

PITTING

.15   O.D.   .15   ENTRANCE

.06   .12

PITTING

GEAR SK13264-2, SER. P1001

PITTING

.02   O.D.   .12

.06   PITTING   .15

GEAR PATTERNS AFTER STONING AND RUN-IN, GEAR SET NUMBER 1

PINION SK13264-1, SER. P1009

.09   O.D.   ENTRANCE

.07   .13

PATTERNS
INCONSISTENT
ON EACH TOOTH

O.D.

O.D.

GEAR SK13264-2, SER. P1001

.06   .07

.04   .05
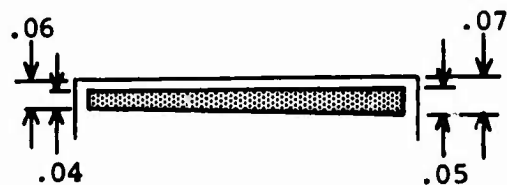
Figure 14.   Tooth Patterns During Load Test (Sheet 2)

PINION SK13264-3, SER. P1005
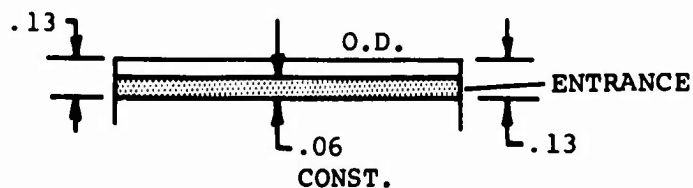
LIGHT LOAD

GEAR SK13264-4, SER. P1003
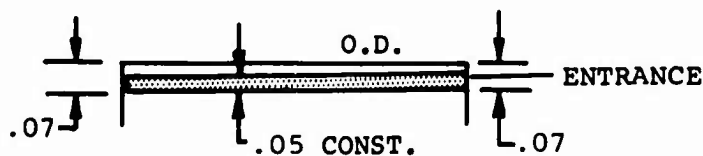
PINION
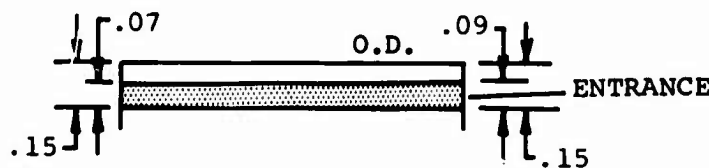
TORQUE
3,000 IN.-LB

GEAR

TORQUE
21,000 IN.-LB

PINION

GEAR

Figure 14.   Tooth Patterns During Load Test (Sheet 3)

PINION SK13264-3, SER. P1005

PITTING

.08   O.D.   .12

ENTRANCE

.14   *PITTING   .15

GEAR SK13264-4, SER. P1003

PITTING

O.D.

ENTRANCE

.08   *PITTING   .10

*SINGLE PITS

GEAR PATTERNS AFTER RUN 2, GEAR SET NUMBER 2

PINION SK13264-3, SER. P1005

O.D.

ENTRANCE

.15   .15

PITTING ●

GEAR SK13264-4, SER. P1003

.09

ENTRANCE

.120
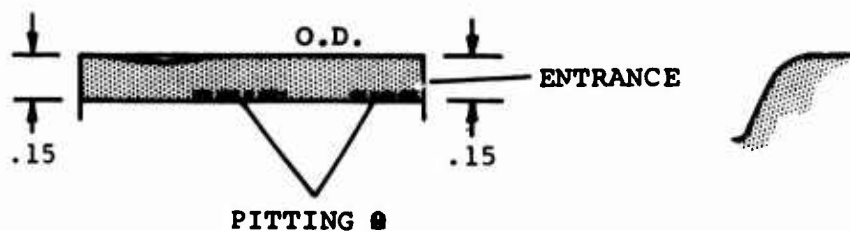
PITTING ●

*PITTING SHOWN AS A COMPOSITE PICTURE; COMBINATIONS
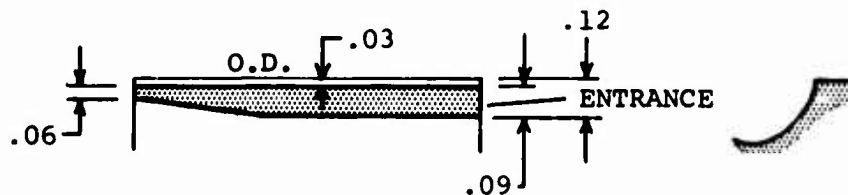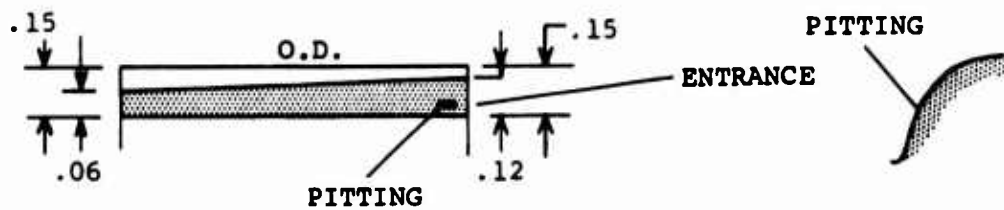OF LOCATIONS SHOWN VARY ON INDIVIDUAL TEETH

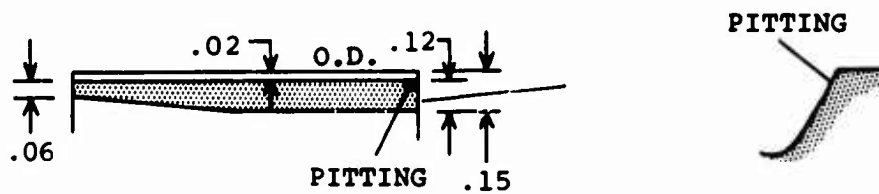Figure 14.   Tooth Patterns During Load Test (Sheet 4)

GEAR PATTERNS AFTER RUN 3, GEAR SET NUMBER 2

PINION SK13264-3, SER. P1005
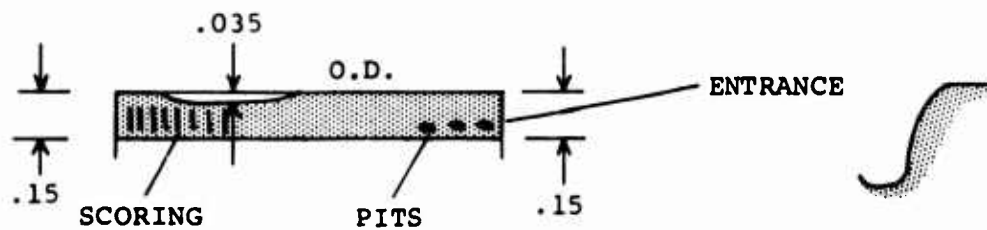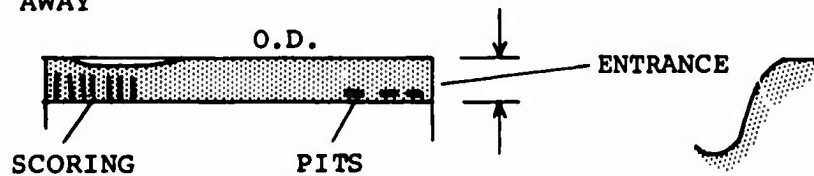
GEAR SK13264-4, SER. P1003
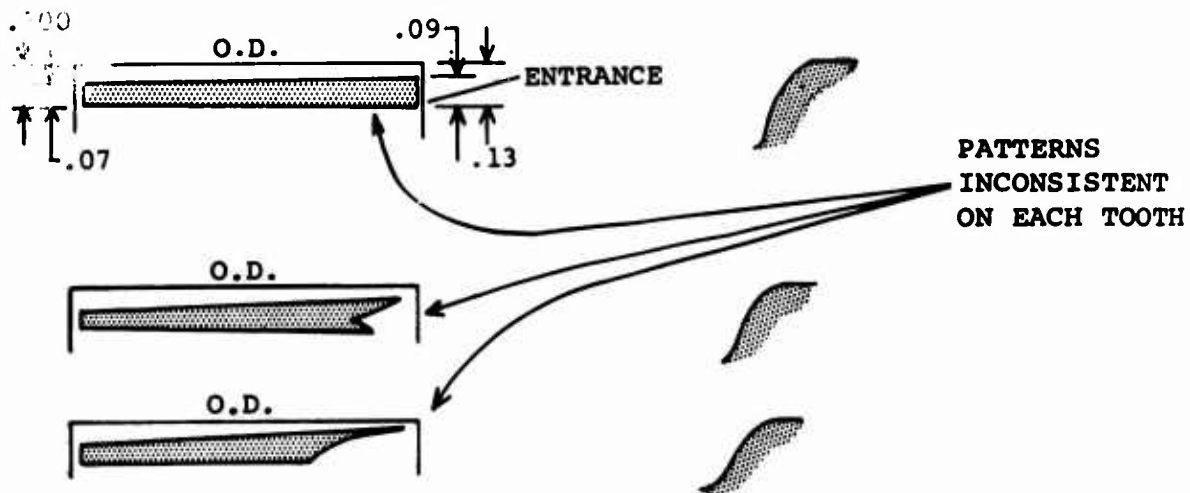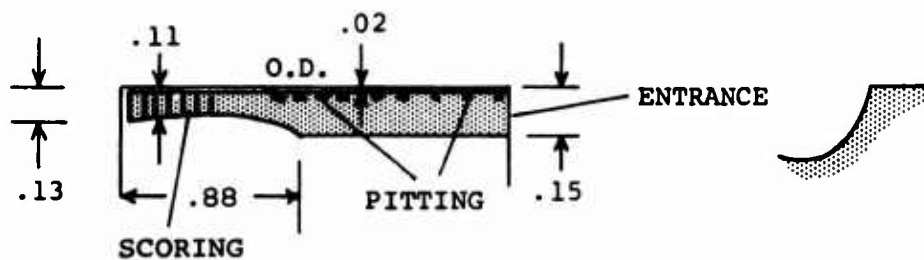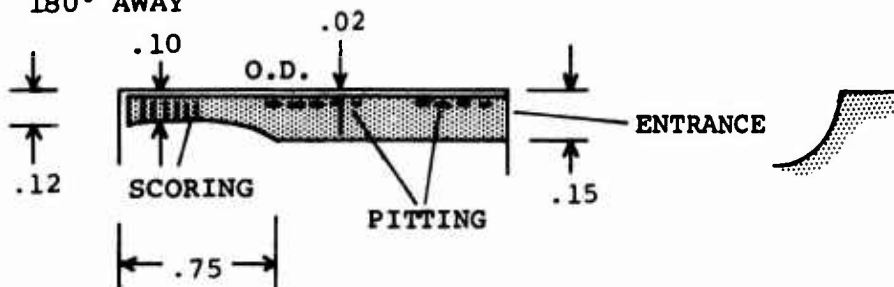

Figure 14.   Tooth Patterns During Load Test (Sheet 5)

below the edge radius. Pitting is generally considered
to be a fatigue phenomenon caused by contact pressure
between mating tooth profiles repeatedly engaging and
disengaging. Magnaglo inspection of both members did
not reveal any crack indications.

The pits were removed from both members by hand stoning
so that the testing could continue. The gear set was then
run in for two more hours (432,000 cycles) to determine
the effect of stoning on the tooth patterns. The subse-
quent visual inspection revealed an inconsistent pattern
on most of the teeth (see Figure 14, Sheet 2).

Test run 3, conducted at 205 percent of the design load leve_,
ended with surface fatigue failure -- severe spalling and
partial tooth breakage -- at 2.98 million cycles. Pitting
became evident early in the run and progressed across the
face width. During the last several hours of this run,
scale changes from 0.5 volt per centimeter to 1.0 and 2.0
volts per centimeter were needed to keep the increased
accelerometer signature on the oscilloscope. Subsequent
magnaglo inspection revealed numerous cracks on both members.

Static Pattern Check and Run-in of Gear Set Number 2

Static pattern checks on gear set number 2 indicated a
tapered tooth pattern similar in inclination and direction
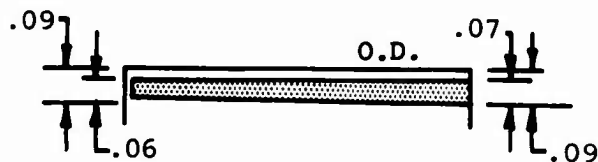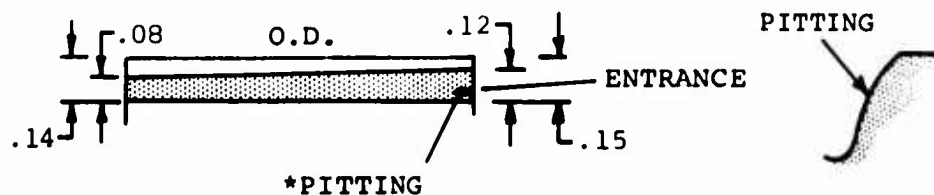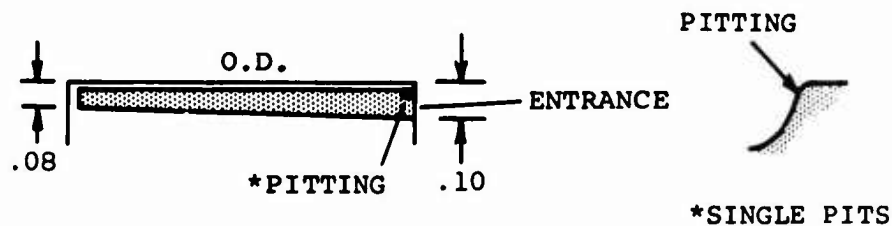to the number 1 gear set, but to a much lesser degree -
approximately 0.02 inch   from end to end (see Figure 14,
Sheet 3). Visual inspection upon completion of the run-in
period did not reveal any adverse conditions.

Load Test Runs of Gear Set Number 2

Test run 1 was completed at 133 percent of design load.
Visual inspection at this time revealed conditions similar
to gear set number   at the completion of test run number
2.   See Figure 14, Sheet 1 for this condition.
The single pit was not as deep or pronounced as the number
1   gear set, and originated at the very edge of both mem-
bers. It should be noted that this gear set had an increased
contact ratio (1.4), increased face width over pitch diameter
ratio, and increased oil supply to the entering side of mesh.
The pitting condition was removed by hand stoning.

Test run 2 was completed at 215 percent of the design
load level. Visual inspection at this time revealed minor
pitting on the gear member and increased pitting on the
pinion member (one tooth in particular). The tooth patterns
after stoning were consistent and therefore not similar to
gear set number 1. See Figure 14, Sheet 4 for this
condition. It was decided to continue running at the next
load level without modification.

Test run 3 was conducted at 250 percent of the design
load level. This test run resulted in failure at 1.08
million cycles due to surface fatigue and failure of one
tooth on the gear member. Subsequent magnaglo inspection
revealed 12 cracked teeth on the gear member and no
indications on the pinion member. Figure 14, Sheet 5
reflects pattern conditions at the conclusion of this run.

The test gearbox was instrumented for a sweeping wave
analyzer during the three load test runs. The analysis of
accelerometer data identified the major vibration frequen-
cies as harmonics of the basic tooth frequency. As the test
progressed, the amplitude of these harmonics increased
and closely associated frequencies appeared.

Figure 15 illustrates the condition of the gear and Figure 16
the pinion at the conclusion of testing.

Metallurgical Findings

A metallurgical analysis of gear set number 2 made upon
completion of testing included:

1. Nital Etch Examination

   Nital etch examination showed no evidence of grinding
   damage.

2. Microhardness Traverse

   Microhardness traverse was made of the failed
   tooth on the gear member. The results are plotted
   in Figure 17. While the top land of the tooth
   showed a highly acceptable hardness gradient, the

40
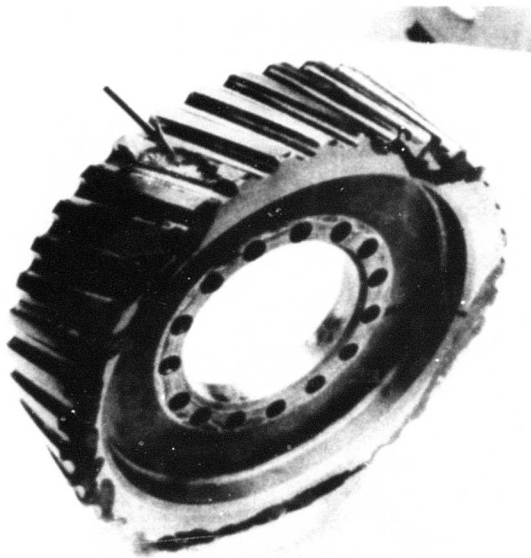
GEAR SHOWING FAILED TOOTH

PITTING EVIDENT ON
TEETH. ARROW POINTS
TO AN INCIPIENT CRACK.
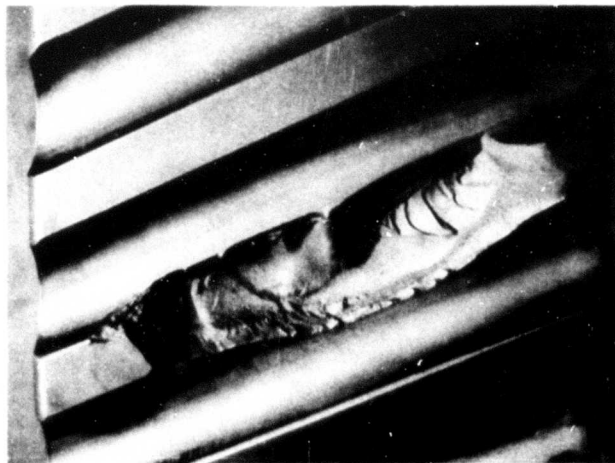
AREA OF TOOTH FAILURE

FAILED SEGMENT

Figure 15. Gear Condition After Final Run, Gear Set
Number 2
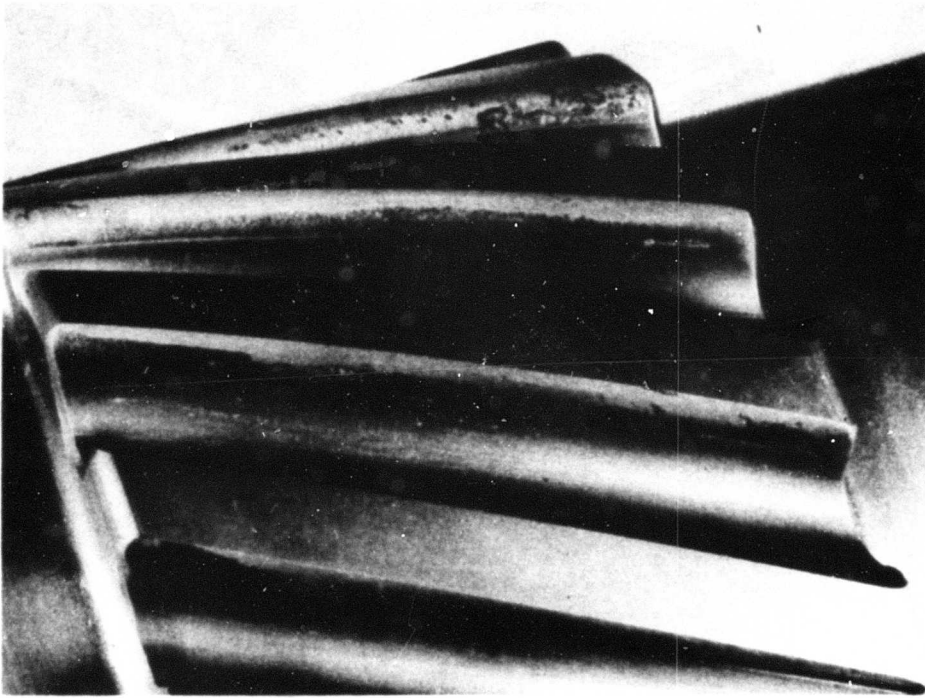
TYPICAL HEAVY FROSTING
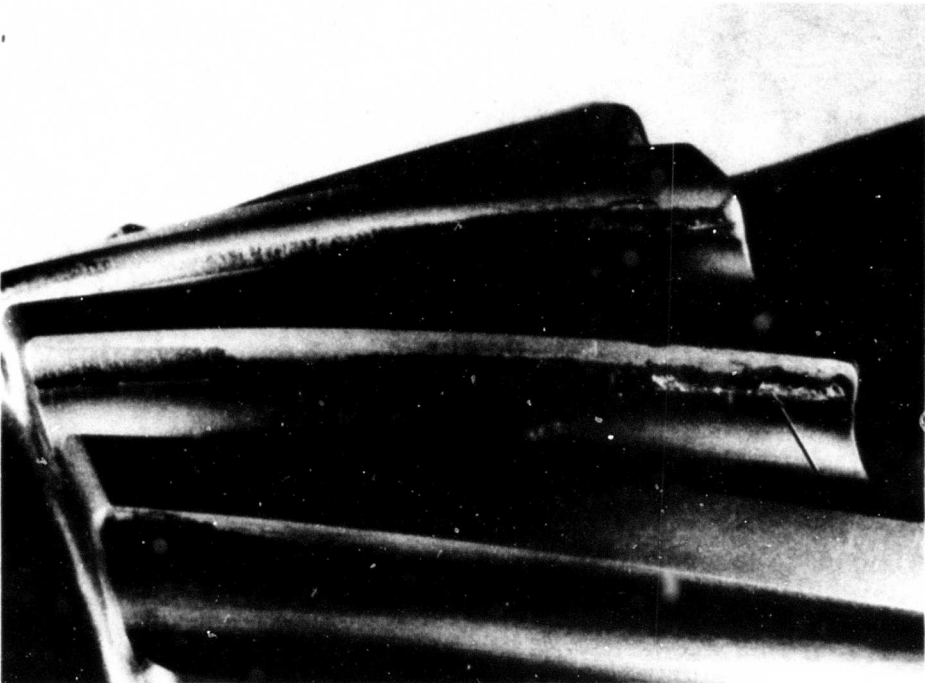
WORST PIT (ENTERING SIDE)

Figure 16.   Pinion Condition After Final Run, Gear Set
            Number 2
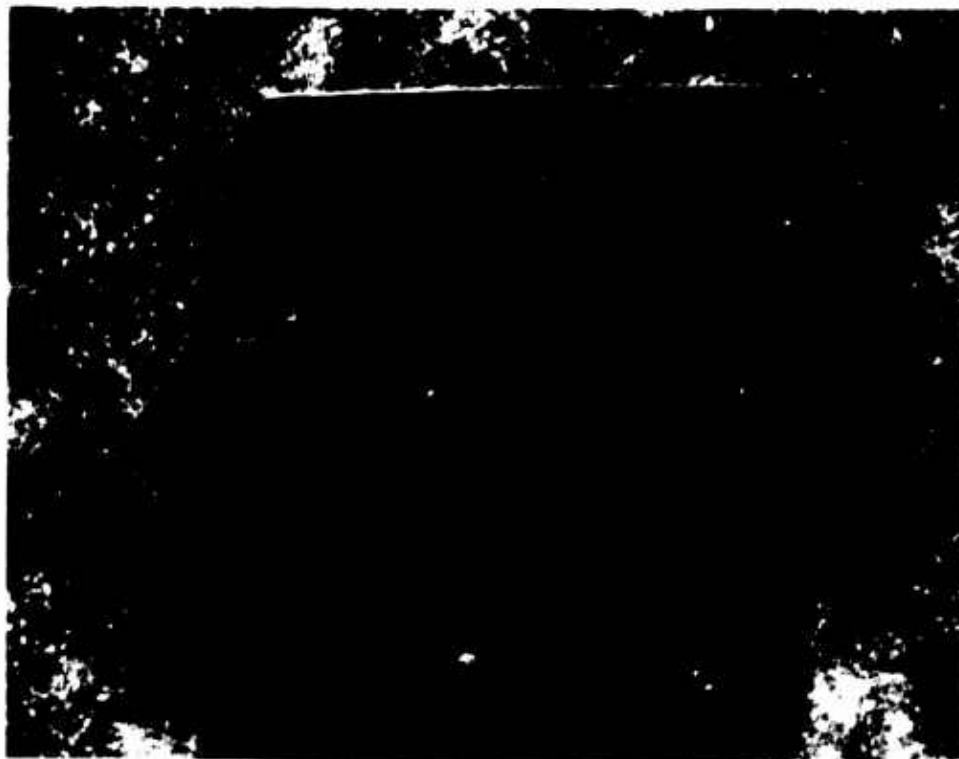
SECTION OF FAILED TOOTH
(ARROW INDICATES EFFECT OF IMPINGEMENT
OF DRIVE-SIDE TOOTH TIP)
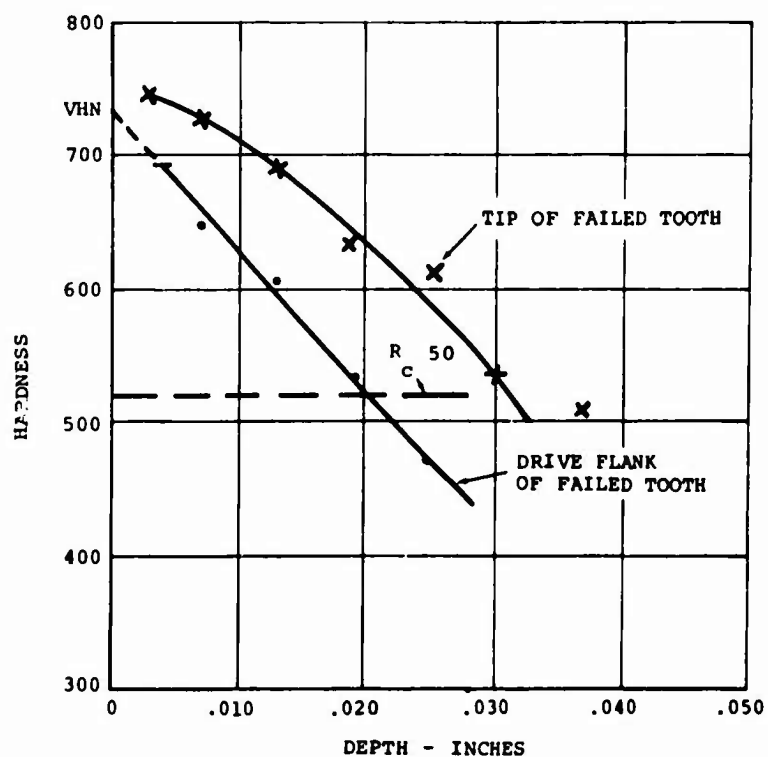
MICROHARDNESS TRAVERSE OF THE ABOVE SECTION

Figure 17. Condition of Gear Set Number 2

drive flank is marginally acceptable: the gradient
is steep, and the minimum specified depth to 50 R_C
(0.025 inch) is not met. The conclusion is that
too much material had been ground from the flank
and this weakened the tooth. Grinding stock re-
moval can be minimized in the future because of the
experience gained from this test series. Preserva-
tion of the carburized layer requires knowledge of
the heat-treat distortion to be expected of this
type of gear and also requires a means of inspecting
the tooth profile after cutting.

3. Microexamination

A microexamination for retained austenite, carbide
network, and cleanliness showed that the material
was satisfactory in all respects.

4. Fractographic Examination

A fractographic examination of the failed tooth
located the origin of the fatigue in the drive
side fillet. The specimen did not provide enough
evidence to tell whether the origin was at the
surface or below the surface.

STRAIN SURVEY

To gain additional understanding of the mechanism of con-
formal contact, a pinion tooth strain survey was performed.
Three consecutive pinion teeth were gaged in such a way
that a record of load sharing could be obtained as the
gear and pinion were rolled through mesh under torque.

Gear set number 2 was used in the strain survey. The test
was conducted in the Vertol research test stand used for
the load run evaluation. Torque was applied in a direction
opposite to that used in load running to apply contact to
the unworn (coast side) tooth flanks.

INSTRUMENTATION

Strain gages of type CX-111 (Budd) were applied to the
tension root fillet of the pinion. Figure 18 shows the
placement of gages and their numeration. Resistance
calibration methods were used to establish a known strain
level in each gage. Gage location from the pinion outside
diameter to gage center was initially held as closely as
possible to 0.180 inch. The gages did not extend onto the
contact surface of the tooth. Eight gages were applied to
the middle tooth at equal spacings of 0.33 inch, with gage
numbers 3 and 10 as close as possible to the tooth ends.
Gages 1 and 2 were applied to the leaving end of the pre-
ceding tooth, at the same spacing between gages. Gages 11
and 12 were similarly applied to the entering end of the
succeeding tooth. During the survey, the gages were
expected to pick up load in numerical ascending and de-
scending order, according to direction of rotation.

Additional information was obtained from a rotary potentiom-
eter connected to the pinion shaft, and from a continuous
record of the torque.

45
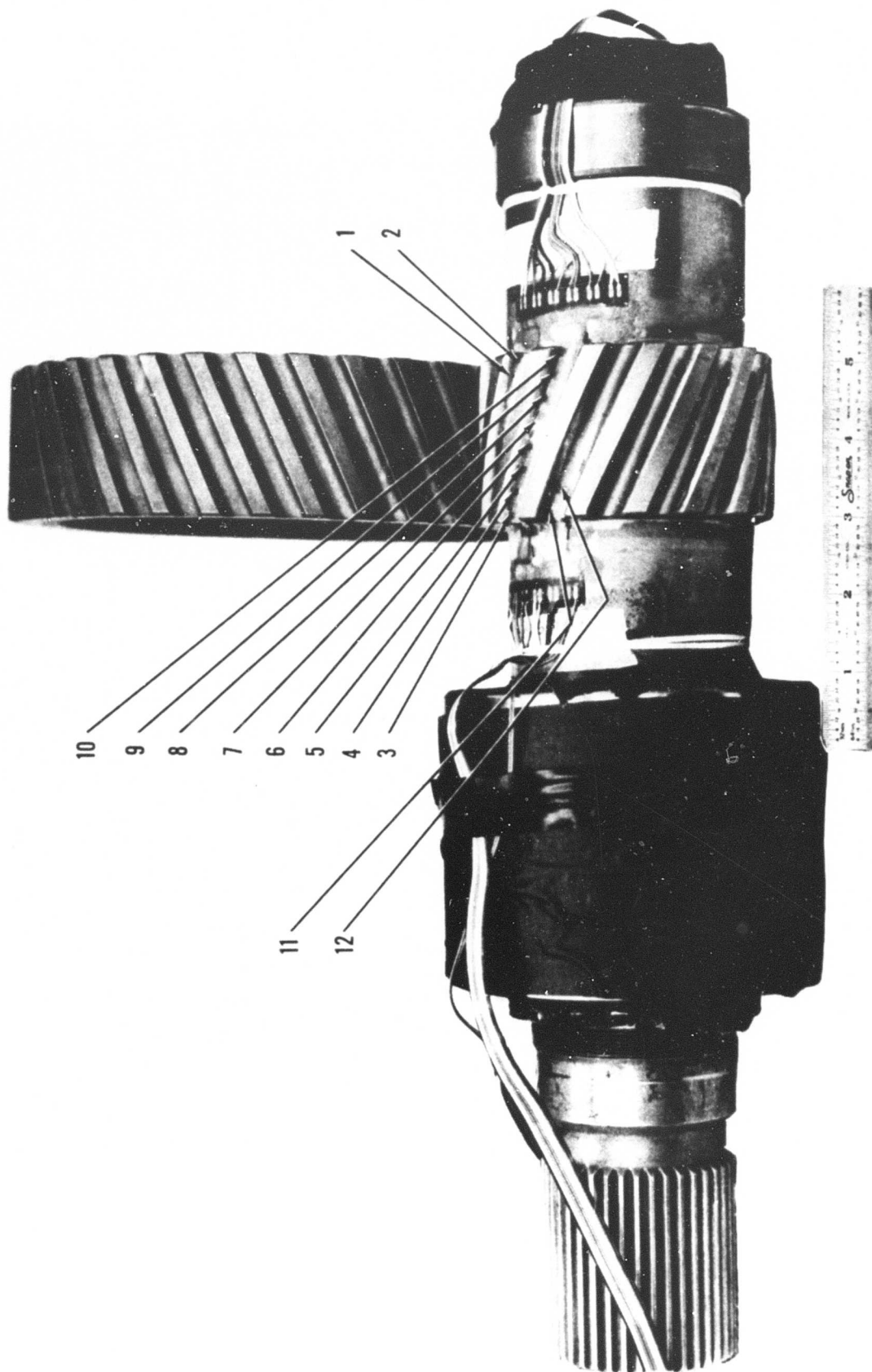
Figure 18.   Instrumented Pinion Showing Gage Numbers

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The effect of the tapered tooth patterns is still more
difficult to measure. Note, however, that gear set number
2, which had little taper, outperformed gear set number 1,
which had a very apparent taper. Gear set number 2 final
surface condition after operation at 16,000 inch pounds,
was markedly better than that of number 1 at the midpoint
of its final run at 13,400. The initial pitting which
occurred on both sets was in the area of the full pattern
(large end of the taper) and is ascribed to lack of lead-
in relief, rather than to pattern deficiency. Lead-in
relief is a correction of the helix angle to improve the
load transfer from one tooth to the next. It is similar
in effect to involute profile modification, in that it
compensates for elastic deflections of one tooth relative
to the next.

Comparison With The Analysis

The analytical method derived for the conformal gear did
not account for maldistribution of load and its effect on
contact stress. The analysis assumed full conformity of
profile, no tip interference, and equal loading from end
to end. The damage observed in the experiment represents
different and less favorable conditions.

When the stress analysis was modified for the reduction in
L_N estimated by the effect of tip interference, the computed
contact stress appears consistent with the observed damage.
The estimate of consistency is based upon involute gear
experience using identical material, surface hardness,
and lubricating oil. The certainty with which contact
deterioration can be related to a stress level may be
less than that relating to a tooth bending failure. Never-
theless, it appears that the present contact stress cal-
culation method is valid, and may be used in assessing the
capacity of future designs.

The possibility of added capacity improvement by other
refinements of the tooth also exists. If these are proven
by test, the present contact stress allowables derived
from involute gearing may be increased, insofar as sliding
velocity due to profile disparaties is reduced. As noted
in the Phase I report, the literature of surface contact
experimentation indicates that a pure rolling contact can
withstand higher pressures without distress than is possible
when sliding is present.

PROCEDURE

The test procedure was to rotate the gear and pinion slowly
through mesh, while observing and recording strain gage
outputs, torque, and angular position. The first roll-
through was with the gear driving. The return to the starting
point was with the pinion driving. The driving member
is the one wherein the loaded side of the tooth is advanc-
ing. A normal speed-decreasing mesh is pinion driving, and
this condition pertained during the load tests.

All information was recorded on a multichannel oscillo-
graph while the gears were being rotated.

RESULTS

A typical record of test results is shown in Figure 19.
Outputs of gages 1 through 12 are shown as they rise and
fall during the mesh cycle. The middle tooth is in mesh
from the first rise of gage 3 to the return to zero of
gage 10. The fall to zero of gage 2 indicates the instant
of disengagement of the first tooth the rise of gage 11
indicates the engagement of the third tooth. The interim
is the period that the middle tooth carries all load. This
single-tooth engagement period is of very short duration.

The left-hand peaks were obtained with the gear driving;
the right-hand peaks were obtained on the return with the pinion
driving. Strain amplitudes during the latter condition were
found to be consistently higher, by an average 100 percent.
The explanation originally devised centered upon tangential
friction forces in the mating teeth. Friction force when
the gear drives is opposite in direction to that when the
pinion drives. This force increases the tensile stress
field at the tooth base when the pinion drives and reduces
it when the gear drives. The attempt was made to measure
this effect experimentally by lubricating the teeth and
comparing the resulting data to the dry state. The results
did not substantiate this explanation. It is possible
that the slow rate of engagement squeezed out most or all
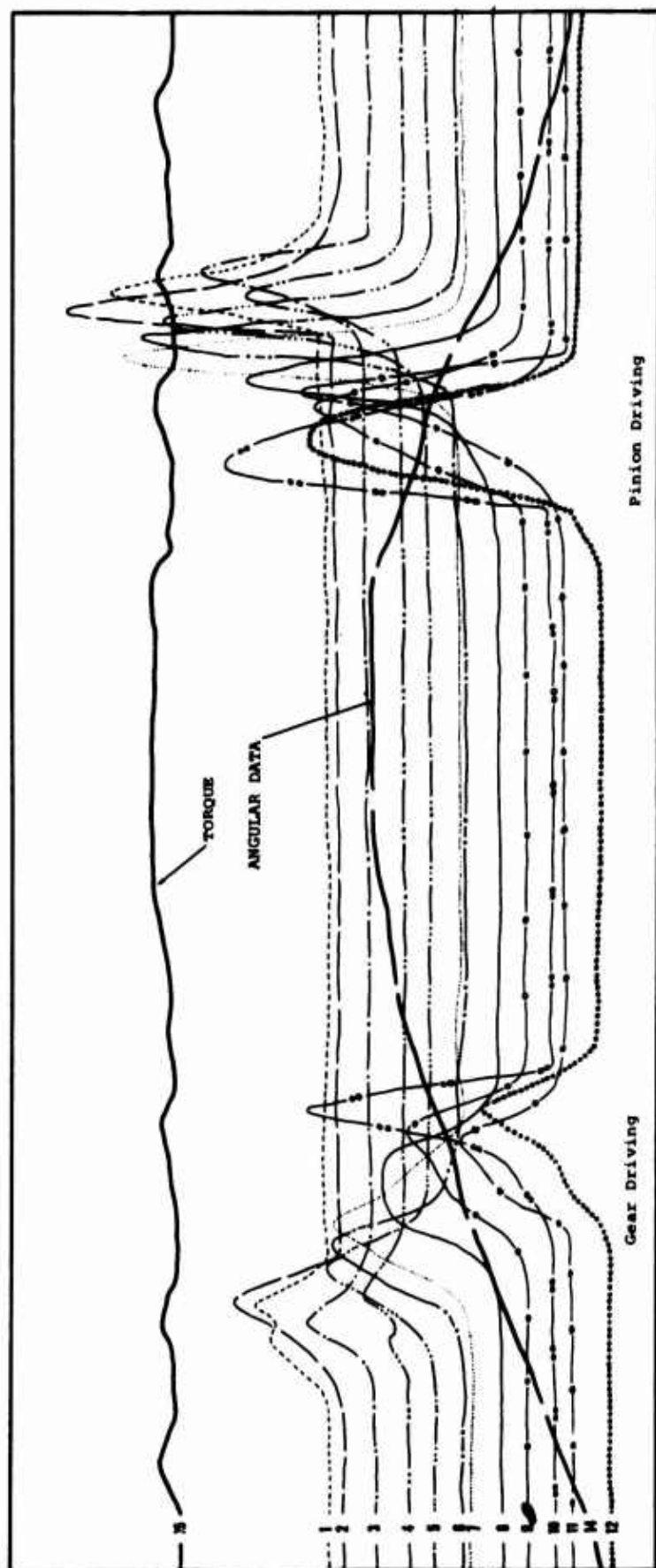
Figure 19. Oscillograph Record of Strain Gage Data

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of the lubricant and, of course, a hydrodynamic effect
was not expected. Therefore, the effect of lubrication
on friction reduction may not have been significant.
Nevertheless, the hypothesis, while not discarded, appears
less probable. At the moment, no other explanation of this
phenomenon has been advanced for study.

Variations were observed in the torque trace. The maximum
variation shown is 3 percent of the total applied torque.
Torque variat'ons did not appear to repeat in a cyclic
pattern and were not clearly identifiable with any of the
rotatinc. components. Maximum strains at each gage across
the pinion face are recorded in Table II. Three test
conditions are shown. The average gage output has been
corrected by the measured position of each gage relative
to the center of the observed band of contact.

TABLE II
STRAIN GAGE DATA

Test (6500 in.-lb torque)	Gage No.	Measured Amplitude (in.)					Strain (micro in./in.)
		Gear Driving	Pinion Driving	Average	Correction Factor	Corrected Average	
Gear Tooth 28 (dry)	1	1.06	1.74	1.40	1.25	1.75	770
	2	1.70	2.10	1.90	1.00	1.90	836
	3	.87	1.50	1.18	1.00	1.18	519
	4	.60	1.47	1.03	1.43	1.47	646
	5	1.30	2.50	1.90	1.00	1.90	836
	6	1.57	3.13	2.35	1.00	2.35	1033
	7	2.00	3.15	2.57	1.00	2.57	1130
	8	1.65	2.43	2.04	1.00	2.04	897
	9	1.20	2.12	1.61	1.00	1.61	708
	10	2.59	3.29	2.94	.83	2.44	1070
	11	1.42	2.23	1.82	1.00	1.82	800
	12	1.22	3.44	2.33	.77	1.80	792
Gear Tooth 32 (dry)	1	1.30	2.08	1.69	1.25	2.10	925
	2	1.14	2.48	1.81	1.00	1.81	796
	3	.80	1.70	1.25	1.00	1.25	550
	4	.55	1.71	1.13	1.43	1.62	713
	5	1.31	2.80	2.05	1.00	2.05	902
	6	1.60	3.33	2.46	1.00	2.46	1080
	7	1.84	3.50	2.67	1.00	2.67	1175
	8	1.34	2.65	2.00	1.00	2.00	880
	9	1.48	2.35	1.91	1.00	1.91	840
	10	2.70	3.30	3.00	.83	2.50	1100
	11	1.50	2.52	2.00	1.00	2.00	880
	12	2.00	3.78	2.89	.77	2.23	980
(Lubed)	1	.78	2.04	1.41	1.25	1.76	775
	2	1.00	2.53	1.76	1.00	1.76	775
	3	.72	1.66	1.19	1.00	1.19	524
	4	.48	1.57	1.02	1.43	1.46	642
	5	1.06	2.65	1.85	1.00	1.85	814
	6	1.10	3.04	2.07	1.00	2.07	910
	7	1.39	3.29	2.33	1.00	2.33	1025
	8	1.12	3.52	2.32	1.00	2.32	1020
	9	1.14	2.02	1.58	1.00	1.58	695
	10	2.42	3.13	2.77	.83	2.30	1010
	11	1.22	2.40	1.81	1.00	1.81	800
	12	1.22	2.50	1.86	.77	1.43	629

EVALUATION OF RESULTS

The load levels sustained by the second conformal gear test
set indicated that its load-carrying capacity was approxi-
mately equal to that of its involute-form counterpart. The
performance of these gears, despite obvious problems, implies
a considerable development potential that remains to be ex-
ploited. If the contact condition can be improved, the con-
formal gear should surpass the involute gear.

CAUSES OF FAILURE

Load testing revealed phenomena which influenced the types of
failure and the load levels at which they occurred:

1. Tapered tooth pattern

2. Initial pitting at the entering end

3. Impingement of the gear tooth against the pinion
   tooth

4. Carburizing depth less than drawing requirement

The tapered pattern, pitting, and tooth impingement caused
premature contact failure by decreasing the contact area
available. Tooth impingement also increased bending stress
in the gear by increasing the bending arm. The shallow
carburized layer reduced material strength.

Tapered Tooth Patterns

The exact effect of the tapered tooth patterns is difficult
to measure. Note, however, that gear set number 2, which had
little taper, outperformed gear set number 1, which has a very
apparent taper. Gear set number 2 final surface condition
after operation at 16,000 inch-pounds was markedly better
than that of number 1 at the midpoint of its final run at
13,400 inch-pounds. The tapered tooth patterns could be
attributed to one or more of the following conditions:

1. Misalignment of the gear case bores

2. Deflection of the shaft under load

3. Lead error

4. Gear faces not perpendicular to shaft axis

5. Non-uniform curvature of tooth profile

The following investigations were made to localize the condition:

1. Before the tests, the gear cases and shafts were inspected by Vertol Division's Quality Control Department. Alignment of the case bores was within the 0.0000-to-0.0002-inch limit. Runout and shoulder perpendicularity of the shafts were well within the drawing limits.

2. During the static pattern checks, shaft deflection and gear face runout were inspected and found to be negligible. The fact that the bottoms of the pinion patterns were parallel to the outside diameters is further indication of parallel location.

3. Lead-error checks of each gear and pinion at the National Broach and Machine Company showed that deviations were within 0.0000 to 0.0001 inch across the entire face.

4. The conformity of the profile radii to the specified form could be inspected only at the end of the tooth, rather than across the face width. Therefore, non-conformity of profile radii must be suspected as a cause of tapered patterns.

Pitting at Entering End

The initial pitting which occurred on both sets was in the area of the full pattern (large end of the taper), and is ascribed to lack of lead-in relief rather than to pattern deficiency. Modification of the helix angle is needed to compensate for the deflection of the teeth as they pass

through the load zone and to improve the load transfer from one tooth to the next. It is similar in effect to involute profile modification, in that it compensates for elastic deflections of one tooth relative to the next.

Impingement of Tooth Tip

The pattern ended abruptly with a hard line on the pinions of both test sets. Apparently, without tip modification the profile-radius mismatch at the high load levels was not sufficient to overcome tip interference. Figure 17 shows the tip damage to the gear member of set number 2. This tip (noted by arrow) impinged the pinion low on the pinion tooth flank and caused high localized contact stresses, as evidenced by the hard line.

COMPARISON WITH INVOLUTE GEARING

Disregarding for the moment the maldistribution of load which influenced the test results, the performance of the conformal gears can be compared to involute gears of equivalent size having the same diameter and face width. Previous tests indicate that equivalent size involute gears would sustain a bending fatigue failure between 30,000 and 40,000 inch-pounds of pinion torque. Before this level was attained, severe surface contact deterioration would have occurred between 14,000 and 16,000 inch-pounds of torque. The load-carrying capacity would be determined by surface contact deterioration.

The conformal gear experienced mild pitting at 8,600 inch-pounds of torque, with increasing severity up to the last test at 16,000 pounds. These load levels are thus comparable to expectations for similar distress on an equivalent size involute gear.

GEAR LOAD PATTERN

Axial Overlap

The overlapping effect of the conformal gear load pattern is shown by Figure 20. This information was taken from a non-lubricated run at design torque (6500 inch-pounds). The rise and fall of the strain gage outputs have been plotted against pinion angular motion. Test variations in angular velocity

have been corrected by scaling the rotary potentiometer trace at each significant peak stress. The basic reference is taken as the angle between the peak of gage 3 and the peak of gage 11. These gages are located at identical points on successive teeth. There is thus 22.5 degrees (360 : 16) of pinion travel required to rotate 11 to the same position as 3.

Overlap is shown as nearly 2.0 compared to the 1.4 calculated for the design. The reason for this large increase is that the analytical method assumes point contact, while actually under load there is area contact. The overlap ratio of the calculation is indicated by peak stress points. Considering these, angular motions between gages 2 and 3, and 10 and 11, are each approximately 20 percent of the total travel from end to end of one tooth (3 to 10). This is equivalent to a 1.4 axial overlap.

Contact Area

Length of the contact footprint has been seen to affect overlap. From Figure 20 it is possible to determine the axial length of the footprint experimentally. The assumption is made that the center of footprint is located above the gage with the maximum reading at that instant. When gage 2 is at maximum intensity, the angular position of the pinion is known. When gage 2 goes to zero, it is known that the contact with that tooth is broken. The angular position at that instant is also known. The difference in angular position between the two points is a measure of the half-length of the footprint (dimension b of the analysis). When angular travel was converted to motion along the tooth and the distance from gage to end-of-tooth was subtracted, the value of b was found be 0.33 inch. Using the analytical solution, b was found to be 0.29 inch. The value of L_N was taken as the height of the contact band observed from the pinion wear pattern. Comparison of the experimental and analytical results indicates that the calculated value for b is substantially correct. If, however, the initial analytical assumption of full tooth profile height for L_N is taken, the footprint length decreases to 60 percent of that found experimentally. The effective overlap decreases as well. However, a compensatory factor appears in that sliding is reduced as point contact is approached. With sliding reduction, there is a lesser tendency for scoring.
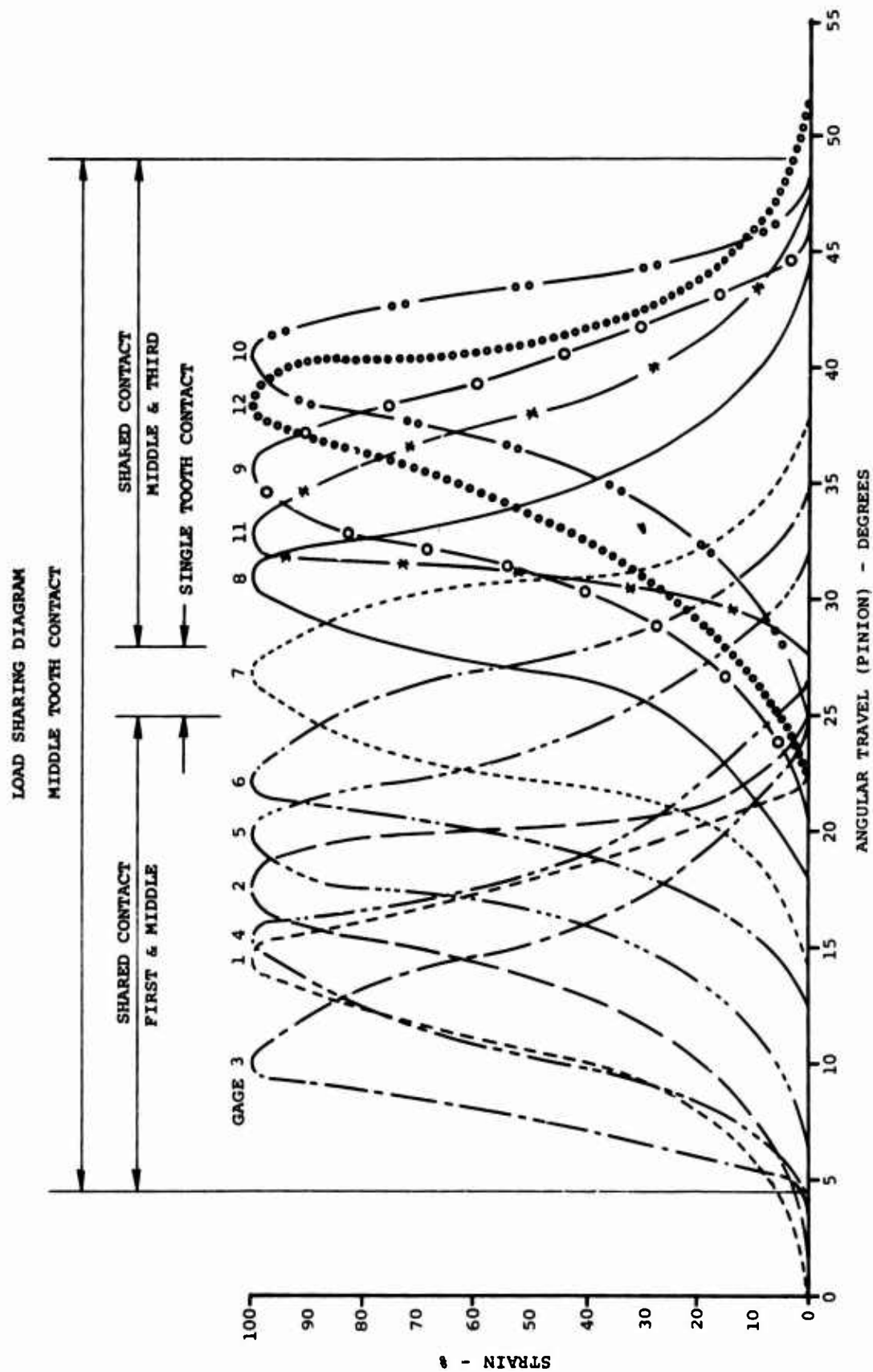
Figure 20. Load Overlapping Effect

STRAIN DISTRIBUTION

Corrected average gage outputs (from Table II) are shown in Figure 21. If gage 10 is discarded, a symmetrical rise to a maximum stress at the tooth center may be visualized. At the center, as has been shown, single tooth contact is supported momentarily. There is no evidence that the tooth end stresses are more critical than the midpoint. Corroboration of this is drawn from the load test results, where failure occurred at the middle of the gear face.

A gage's output may be affected by variables such as gage distance, inclination of the gage axis to the vertical, and bonding of the gage to the specimen. Comparison of stresses between individual gages should therefore be made with some caution. For instance, the readings from gage 10, unsupported by neighboring gages, are treated with less confidence than the trends shown between gages 3, 4, 5, 6, and 7.

COMPARISON WITH CALCULATED STRESS

The original analytical method derived for the conformal gear did not account for maldistribution of load and its effect on contact stress. The analysis assumed full conformity of profile, no tip interference, and equal loading from end to end. The damage observed in the experiment represents different and less favorable conditions.

Contact Stress

Contact stress was increased by tip interference. It is reasonable to suppose that the height of the face (L_N) was effectively reduced to one-half that of the theoretical during the tests. By substitution in the analysis, contact stress would thereby increase 40 percent, and load capacity would decrease by one-half as compared to full-conformity contact. Contact stress computed from the reduced L_N appears to be consistent with the observed damage. This judgement is based upon involute gear experience using identical material, surface hardness, and lubricating oil. The certainty with which contact deterioration can be related to a stress level may be less than that relating to a tooth bending failure. Nevertheless, it appears that the present contact stress calculation method is valid and that current gear stress allowables may be used in assessing the capacity of conformal designs.
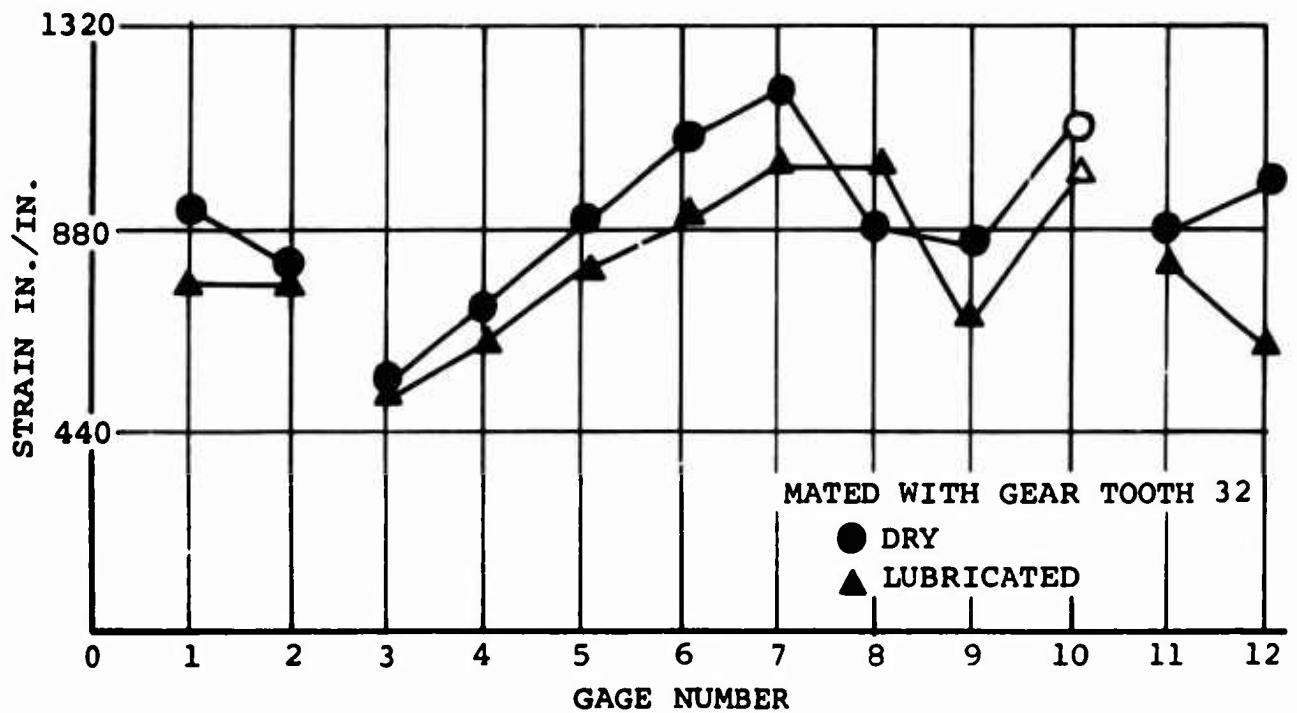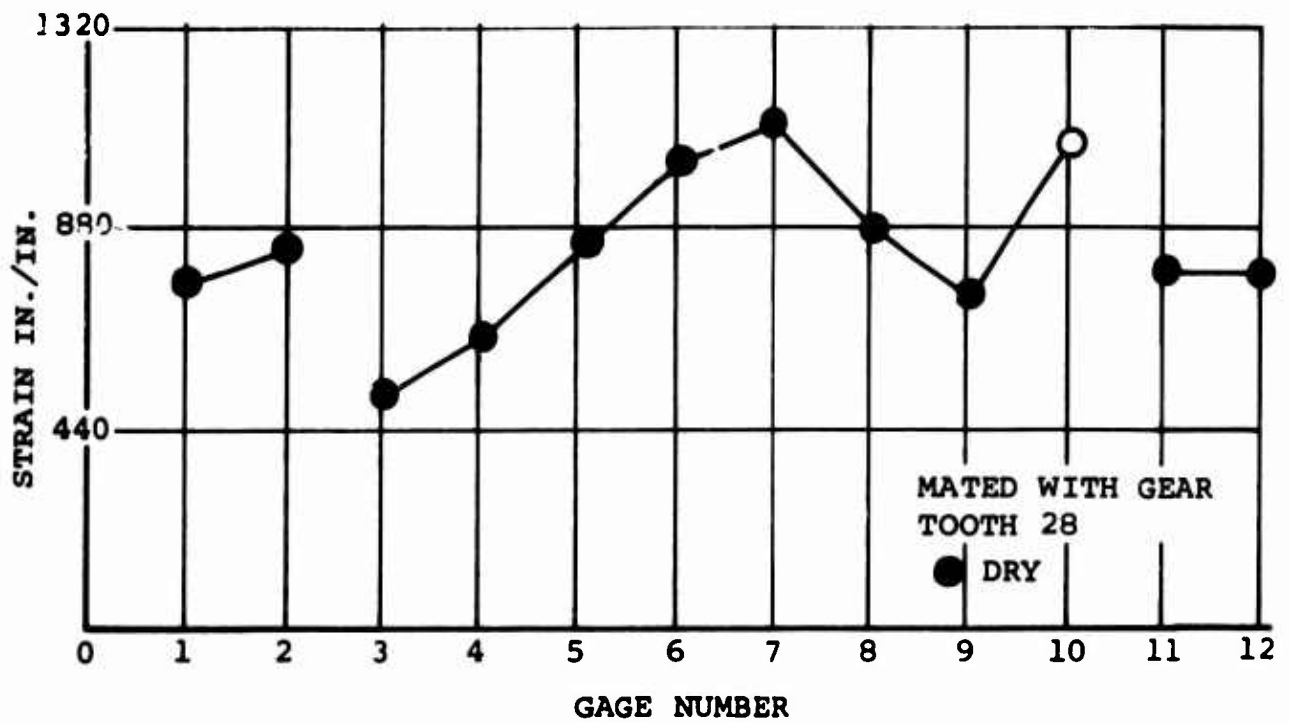
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Figure 21. Maximum Strain Readings (Data From Table II)

The possibility of added capacity improvement by other refinements of the tooth also exists. If these are proven by test, the present contact stress allowables derived from involute gearing may be increased, as sliding velocity due to profile disparities is reduced. As noted in the Phase I report, the literature of surface contact experimentation indicates that a pure rolling contact can withstand higher pressures without distress than is possible with sliding contact.

Bending Stress

Bending strength analysis was not checked by the load-run results, since bending strength was calculated only for the convex tooth pinion and no representative failure of the pinion was obtained. At the failure loads, the calculated bending stress was not high enough to indicate a pinion tooth failure, and such a failure did not occur. The initial estimates of comparative load capacity between conformal and involute gears were based upon pinion tooth strength. This was also the basis for indicating the high face-to-diameter ratio requirement for conformal gear superiority, at equal bending stresses. Rating the capacity upon pinion tooth strength does not appear entirely realistic and may unduly prejudice the conformal gear.

Examination of the load-test specimens indicates that tip impingement increased bending stress in the concave tooth gear by raising the tangential load point above the theoretical mid-face. A compensatory factor is that such a concentrated tip load would increase the axial length of contact (dimension 2b). However, the formulae indicate that the change in height outweighed the reduction in moment factor (K_i). The combined effect was to increase calculated gear bending stress 50 to 75 percent.

In the initial analysis, it has been estimated from photoelastic observation that the gear is the more highly stressed, and in the tests the gear proved to be the weaker member. Because of the peculiar load distribution noted during the test, the failure of the gear is not conclusive proof of the photoelastic estimates; but if testing of improved load patterns continues to fail the gears rather than the pinions, the stress analysis should be revised to calculate gear bending.

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Balancing Contact and Bending Stresses

If continued testing with improved contact patterns indicates that the conformal gear is limited by surface durability rather than by tooth failure, the initial design tendency toward high bending strength may be reversed. A revised balance of bending and contact stresses can be attained by lowering the helix angle. A lowered helix angle increases the axial radius (RE) but also decreases tooth thickness. The result is to increase contact capacity while simultaneously raising bending stresses. Increased bending stress may be acceptable for the following reasons:

1. Strain survey has shown that tooth ends are not overloaded, relative to the tooth center.

2. Modern gear materials and processes are believed to be capable of sustaining higher bending fatigue stresses than are presently used in aircraft practice.

CONCLUSIONS AND RECOMMENDATIONS

The conformal test gears demonstrated load-carrying ability equivalent to that expected of aircraft involute gears. At the failure load level, contact stresses correlated with the analytical method developed in Phase I. The final failure of the conformal test gears was induced by surface deterioration, aggravated by load concentrations from tip interference and lack of helix angle modification. There was also a tapering of the wear pattern under load, the reason for which is not completely understood.

It is likely that load-carrying capacity can be increased more than 100 percent if satisfactory distribution of load can be maintained. Load distribution can be improved by modifying helix angle and tooth profile to eliminate tip interference, provide lead-in relief, and equalize wear patterns. The following modifications are indicated:

1. Helix angle adjustment to equalize wear across the face.

2. Modification of the basic tooth profile to obtain a centralized pattern on the height of the tooth and to make tip impingement less probable.

The increased load capacity expected from these modifications will improve on the specific-weight ratios predicted by the analysis.

Inspection techniques and the equipment now available do not provide accurate measurements along the entire width of the conformal tooth face. New techniques and equipment should be developed to keep pace with the advances which the conformal tooth form represents. It will be necessary to measure accurately the profile of at least one member, and preferably both. Given this, present manufacturing techniques should be fully capable of producing any desired conformal tooth form, and modifications thereof, in high-strength carburized gear steels.

It is recommended that the conformal tooth form be modified to reflect the test results, and that the load-carrying capacity of the improved tooth form then be determined by testing.

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APPENDIX
INSPECTION OF CONFORMAL ARC PROFILE

The method for inspecting the conformal gear circular-arc (convex) pinion tooth profile on an involute checking instrument consists of the following steps:

1. By calculation, determine a master base circle diameter which would produce an involute curve most nearly approximating the circular arc of the convex pinion tooth profile (see Figure 22).

2. Establish and plot the master profile curve (see Figure 23) by calculating the deviation of the conformal circular arc profile from the involute profile determined in Step 1.

3. Check the conformal circular arc profile on an involute checking instrument by setting the instrument to the master base radius determined in Step 1 and record the results.

4. Compose the inspected profile per Step 3 to the master profile determined in Step 2.

Deviations of the inspected profile chart from the master profile chart represent errors in the pinion circular arc profile.

Method for calculating the master base circle radius and involute and the deviations of the conformal pinion circular arc profile from the master involute curve.

1. Calculate master base circle radius (R_bj):

$$R_b = \frac{R_{SAP}}{\left[\left(\frac{E \circ SAP}{57.3}\right)^2 + 1\right]^{\frac{1}{2}}}$$

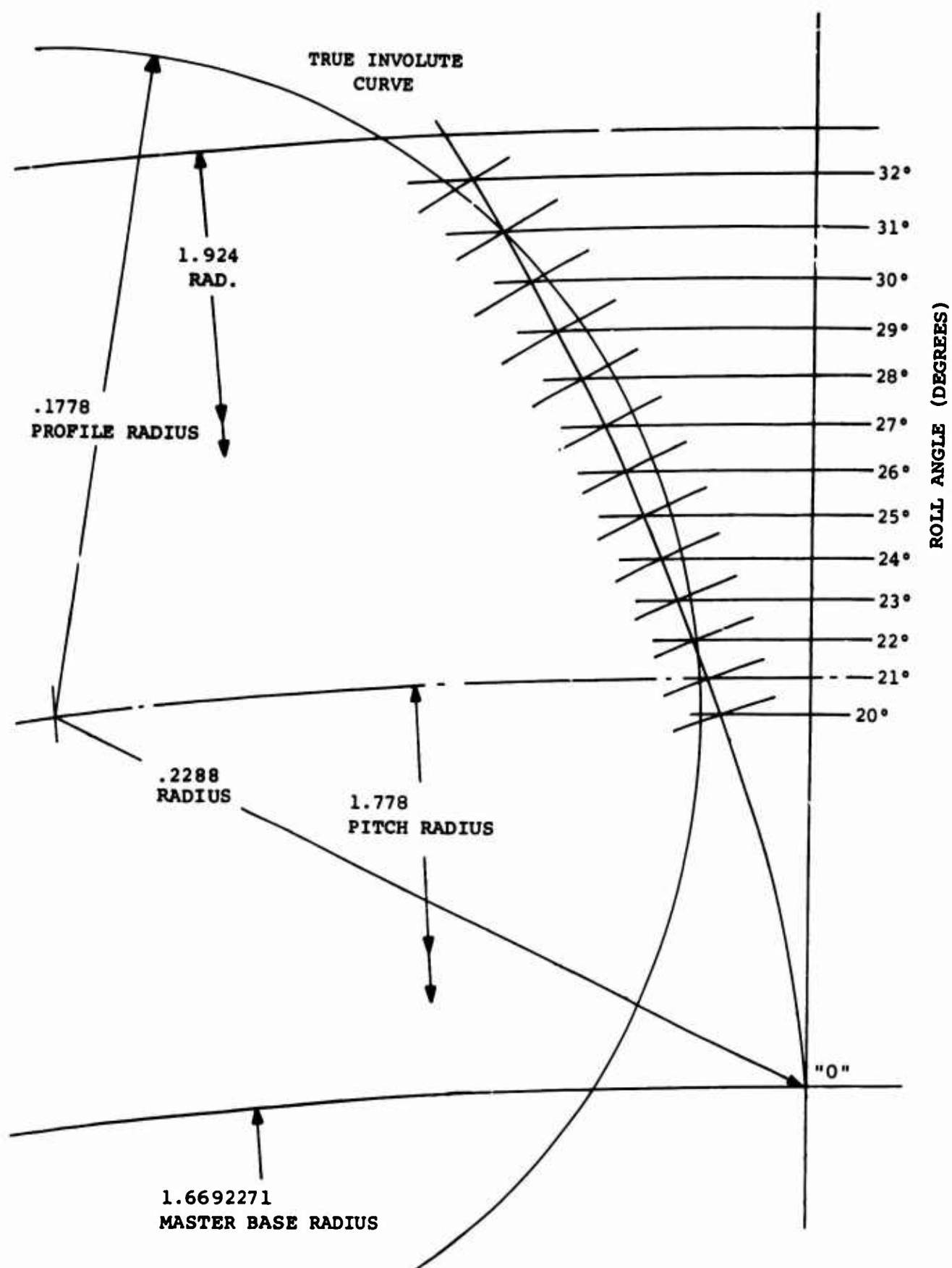$$R_b' = Rm \times \cos \phi$$

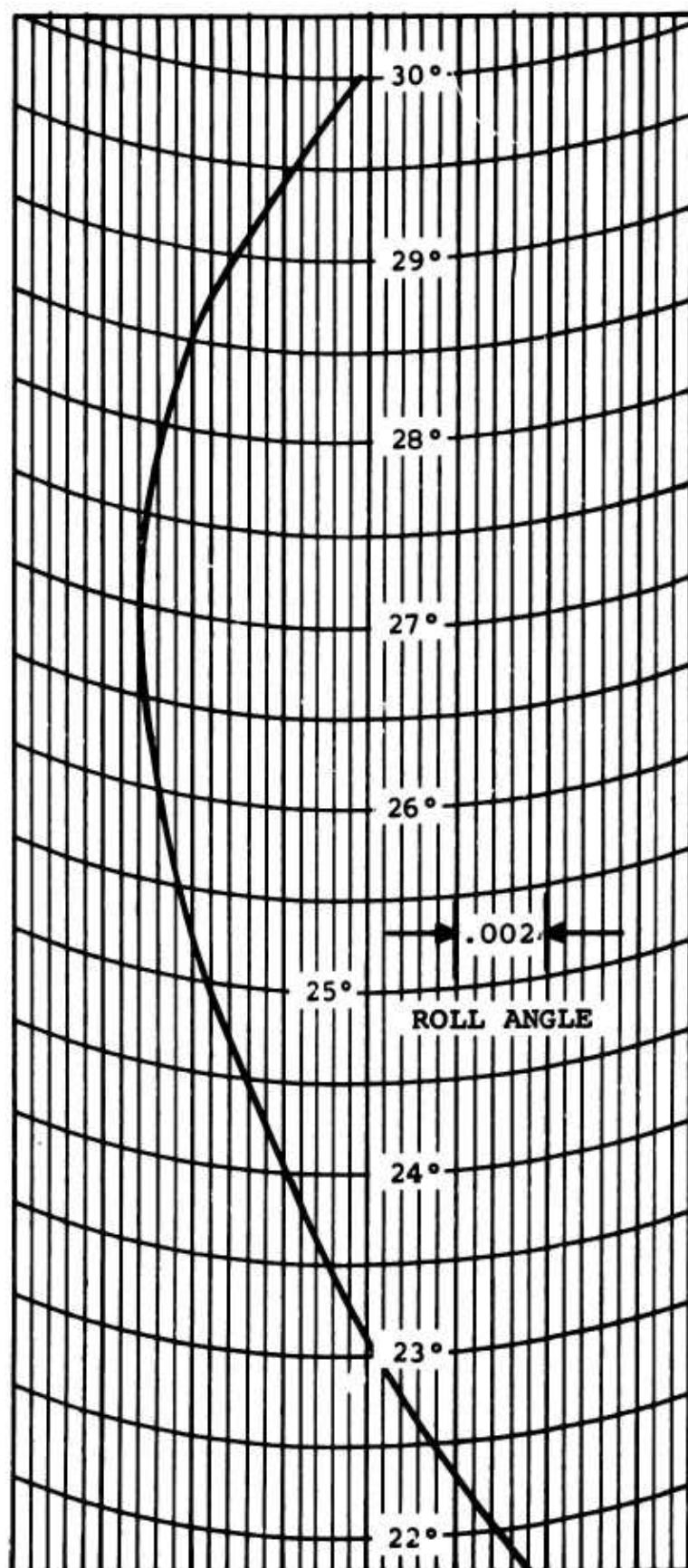$$Rm = \frac{R_{SAP} + Ro}{2}$$

Figure 22. Involute Curve Construction

Figure 23.   Master Profile Curve

$$\cos \theta = \frac{(Rm)^2 + (r)^2 - (R_1)^2}{2(Rm) \times r}$$

$$\emptyset = 90° - \theta \quad \text{(Round off } \theta \text{ to nearest degree)}$$

$$E°_{SAP} = 57.3 \left[\left(\frac{R_{SAP}}{R_b'}\right)^2 - 1 \right]^{\frac{1}{2}}$$

(Round off $E°_{SAP}$ to nearest degree)

2. Calculate deviations of circular-arc profile from master involute curve (Δ).

$$\Delta = R_1 \, SIN \, (E° - \phi°) +$$
$$\sqrt{r^2 - \left[R_b - R_1 \, \cos (E° - \phi°)\right]^2} - R_b \times \overline{E}$$
$$\cos \phi = \frac{(R_1)^2 + (R_b)^2 - (r')^2}{2 \, R_1 \, (R_b)}$$

By layout (30X scale) establish a radius (r') that will rotate the roll to be checked, which represents the conformal pinion profile, into a position that will result in a desirable deviation from the master involute.

Where

R_b	is	master base circle radius
Δ	is	deviation of circular arc profile from master involute curve
R_1	is	radius to center of conformal tooth profile arc
r	is	radius of conformal tooth profile arc
R_{SAP}	is	radius to start of active profile
Ro	is	radius to outside diameter
$E°$	is	roll angle to selected point on master involute

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EXAMPLE

Given:

$$R_1 = 1.778$$

$$r = 0.1778$$

$$R_{SAP} = 1.768$$

$$Ro = 1.924$$

Determine:

R_b = master base circle radius

Δ = deviation of circular arc from master involute

1. $$Rm = \frac{R_{SAP} + Ro}{2} = \frac{1.768 + 1.924}{2} = 1.846$$

$$\cos \theta = \frac{(Rm)^2 + (r)^2 - (R_1)^2}{2 \, (Rm) \times r} = \frac{.2780448}{.6564376} = .4235662$$

$$\theta° = 65° \ (\text{Rounded off})$$

$$\emptyset = 90° - \theta = 90° - 65° = 25°$$

$$R_b' = Rm \times \cos \emptyset = 1.846 \times .9063078 = 1.6730442$$

$$E°_{SAP} = 57.3 \left[\left(\frac{R_{SAP}}{R_b'} \right)^2 - 1 \right]^{\frac{1}{2}} = 57.3 \left[\left(\frac{1.768}{1.6730442} \right)^2 - 1 \right]^{\frac{1}{2}}$$

$$= 20° \ (\text{Rounded off})$$

$$R_b = \frac{R_{SAP}}{\left[\left(\frac{E°_{SAP}}{57.3} \right)^2 + 1 \right]^{\frac{1}{2}}} = \frac{1.768}{\left[\left(\frac{20°}{57.3} \right)^2 + 1 \right]^{\frac{1}{2}}}$$

$$= 1.6692271$$

2. $r' = 0.2288$ (by 30X layout)

$$\cos \phi = \frac{\left(R_1\right)2 + (R_b)2 - (r')2}{2 R_1 (R_b)}$$

$$= \frac{3.1612840 + 2.7863191 - .0523494}{5.9357716}$$

$$= .9931739$$

$$\phi = 6.70°$$

$$\Delta = R_1 \text{ SIN } (E° - \phi°) +$$
$$\sqrt{r^2 - \left[R_b - R_1 \cos (E° - \phi°)\right]^2} - R_b \times \overline{E}$$

$$= 1.778 \times .2300772 +$$
$$\sqrt{.1778^2 - \left[1.6692271 - 1.7303005\right]^2} -$$
$$1.6692271 \times .3490658$$

$$= .4090773 + \sqrt{.0316128 - .0037300} - .5826700$$

$$= .4090773 + .1669814 - .5826700$$

$$= - .0066113 \text{ (at } 20° \text{ of roll)}$$

The calculation of Δ is repeated for all desired degrees of roll. The results for this example are as follows:

Degrees of Roll $E°$	Amount of Deviation Δ
20	− .0066113
21	− .0030846
22	+ .0000926
23	+ .0028479
24	+ .0051059
25	+ .0067880
26	+ .0078111
27	+ .0080860
28	+ .0075152
29	+ .0059899
30	+ .0033863
31	− .0004407
32	− .0056666

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13. ABSTRACT

The conformal gear has been studied as a means of improving helicopter final reduction drives. By eliminating certain inherent limitations of the involute gear, the developed conformal tooth form has the potential of bettering current power-to-weight ratios. Under USAAVLABS sponsorship, the Vertol Division of Boeing has conducted an analytical and experimental effort on carburized and ground conformal gears which has resulted in the first published work in this area. (Earlier efforts had used unground, commercial gears.) This report concludes Phase II of the study conducted under Contract DA44-177-AMC-101(T) Modification 1. The study was initiated upon receipt of the contract modification on 17 June 1964.

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